

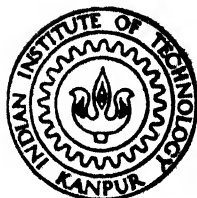
# RECONSTRUCTION TOMOGRAPHY USING CHORD-SEGMENT-INVERSION TECHNIQUE

by

R. K. JARWAL

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NUCLEAR ENGINEERING AND TECHNOLOGY PROGRAMME

INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

APRIL, 1987

# **RECONSTRUCTION TOMOGRAPHY USING CHORD-SEGMENT-INVERSION TECHNIQUE**

**A Thesis Submitted  
In Partial Fulfilment of the Requirements  
for the Degree of**

**MASTER OF TECHNOLOGY**



by

**R K. JARWAL**

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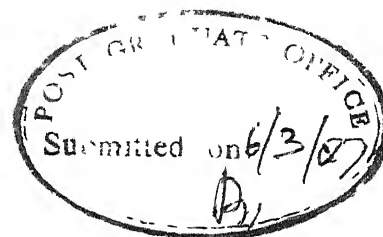
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CERTIFICATE

This is to certify that this work on "Reconstruction Tomography Using Chord-Segment-Inversion Technique" by Mr. R.K. JARWAL has been carried out under our supervision and has not been submitted elsewhere for the award of a degree.

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ABSTRACT

Computerized Tomography (CT) has been demonstrated to be a good technique for measuring point-density (void-fraction) in two-phase flow systems. Recently, improvements have been suggested regarding the choice of filter functions in CT methods. These methods are based, essentially, on the discrete implementation of the Radon Inversion Formulae which is widely used in the medical imaging area. Such methods do not require any information, a priori, regarding the distribution of the density (or the void-fraction).

A very simple method involving the tomographic chord-segment inversion, has been developed and tested for two-phase flows having radially symmetric density distributions. This method is much simpler and consumes less CPU time relative to the more general methods of tomographic reconstruction. For test functions the reconstructed density distributions are almost exact. For an air-water bubbly flow data the reconstructed values have a maximum deviation of  $\pm 0.03 \text{ g/cm}^3$ . The range of investigation of the air-water flow data was  $0.6 - 0.9 \text{ g/cm}^3$ , i.e. void-fraction range of 40% to 10%. These results are comparable to the results obtained by the more general methods based on the Radon Inversion Formulae.

## CHAPTER-1

### INTRODUCTION

Computer Aided Tomography (CAT) is being widely used in the medical area for the diagnosis of various cancerous tissues. The methodology incorporates scanning of the patient with gamma rays using appropriate tomographic algorithms to reconstruct the density distribution of tissues [1]. A basic form of CAT was used in Japan in 1946 named "Rotation Radiography". In this method the patient was placed on a rotatographic table, X-ray tube and film were rotated around the patient from  $0^{\circ}$ - $360^{\circ}$  while the pictures were taken and collected information regarding various cross sections in the range  $0^{\circ}$ - $360^{\circ}$ .

This concept of measuring density distribution was first investigated by Schlosser et al [2] for a two-phase air-water system. The results obtained in void fraction/density measurements have been summarized by Kulacki et al [3]. The technique for measuring density distribution has great significance because accurate measurements of density for various flow systems and components in nuclear systems facilitates the computation of heat transfer rates. This information is vital from the reactor safety view-point because it helps in predicting core burnout etc. This technique can be applied in other fields like chemical

industries, food-processing and several other research areas.

The various reconstruction methods can be broadly classified in the following categories;

- (a) Series Expansion Methods
- (b) Transform Methods.

In the Series Expansion Methods (SEMs) the pixel-wise distribution of the function (under-investigation) is assumed and then suitable iterative and noniterative procedures are applied to achieve the reconstruction of the function in the region of interest [4]. The iterative SEMs are Algebraic Reconstruction Technique (ART), Simultaneous Iterative Reconstruction Technique (SIRT), etc. The non-iterative SEMs are Angular Harmonic Decomposition (AHD) and Polynomial Decomposition (PD).

The transform methods are based on the analytic formulas based on the Radon Inversion Technique. The transform methods are of two types [5]:

- (a) Direct Fourier Inversion (DFI)
- (b) Convolution-Back-Projection (CBP).

In DFI method the direct Fourier transform of the projected data is taken and subsequent 2-d Fourier inversion leads to the reconstruction of the unknown distribution. In CBP method the data is convolved with a suitable filter

function and then back projection of the convolved data results in the reconstruction of the unknown distribution.

An important feature of the tomographic methods is that the point-density measurements can be made in a non-invasive manner without any prior knowledge of density distribution. In non-invasive methods the measurements are taken in such a way that the system does not get affected which is the case with tomographic methods.

The currently established reconstruction methods are mathematically and computationally complex, so the present work is an attempt to develop a simple algorithm to measure point densities in radially symmetric flow distributions. Such patterns are often encountered in gas-liquid flows through pipes. This chord-segment-inversion (CSI) algorithm has been demonstrated to be an extremely efficient method with further processing resulting in the radial density maps. The algorithm has been tested against some simulated radially symmetric distributions representing bubbly and annular flow distributions.

Additionally the CSI method has been applied to reconstruct density map for the air-water bubbly flow data [2,3,6]. The results appear to be comparable with the earlier known more complex tomographic methods [3,7,8]. The CPU time for the CSI technique is much less than that in other general tomographic methods.



## CHAPTER 2

### THEORETICAL FORMULATION

In this chapter a brief explanation of how the absorption coefficient is related to the source strength and detector reading along a particular chord is given. The geometry under consideration is the fan-beam geometry. The discrete form of this relationship has been explained and the chord-segment matrix and its triangularization has been introduced.

#### 2.1 PRELIMINARIES

The single-beam radiation attenuation phenomenon is represented by

$$N = N_0 \exp \left[ - \int_c \mu(r, \emptyset) ds \right] \quad (1)$$

where,

$N$  = detector reading (count/second)

$N_0$  = source strength (counts/second)

$s$  = path of radiation (ray)

$c$  = chord along which  $s$  is integrated

$\mu$  = absorption coefficient

$r, \emptyset$  = cylindrical coordinates.

Rewriting Eqn.(1), we have

$$d = \int_C \mu(r, \emptyset) ds \quad (2)$$

where,

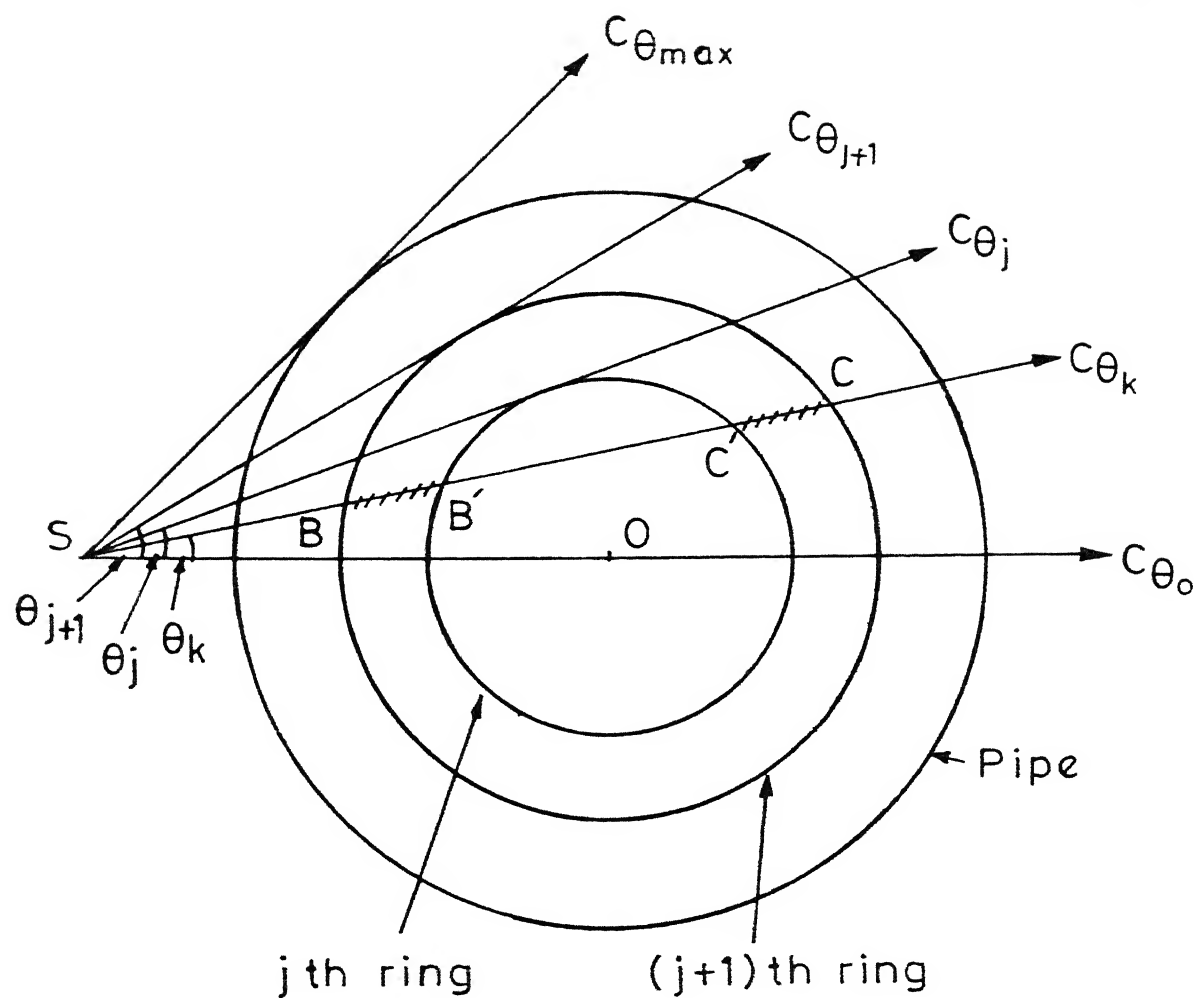
$$d = \ln (N_o/N) \quad (3)$$

In the fan-beam geometry (i.e. the geometry in which the beam diverges from the source), for a particular chord (ray),  $C_\Theta$ , corresponding to the ray making an angle  $\Theta$  from OS line (See Fig.1), the data is denoted by  $d_\Theta$ .

Thus,

$$d_\Theta = \int_{C_\Theta} \mu(r, \emptyset) ds \quad (4)$$

Here,  $d_\Theta$ , for different values of  $\Theta$ , is the data to be processed by the tomographic algorithms (in our case the tomographic algorithm is CSI). The reconstruction of the absorption coefficient, is done and the density,  $\langle \rho \rangle$ , (or void-fraction  $\langle \alpha \rangle$ ) is determined by a calibration of the "CT numbers" (in this case  $\mu$ ) using some known density distributions. The reconstruction has been done for some known  $\mu$  like 1,  $r$ ,  $e^x$ ,  $e^{-x}$  (See Fig.2). In other words, if a set of data  $d_\Theta$  for any density distribution is known to us then we can obtain the CT numbers for that distribution and hence by calibration the density.



$$\text{-----} \rightarrow S_{k,j} = BC - B'C'$$

$S$  - Source

$O$  - Object centre

$SO = D$

Fig.1 Data collection geometry

The discrete form of Eqn.(2) can be written as (Fig.1),

$$d_k = \sum_{j=1}^m S_{k,j} \mu_j, \quad k = 1, 2, \dots, m \quad (5)$$

where,

$S_{k,j}$  = length of the segment of the  $k$ th ray  
falling in the  $j$ th ring

(The hatched lines in Fig.1)

$$= BC - B'C'$$

$\mu_j$  = average value of  $\mu$  in the  $j$ th ring  $\mu$

$m$  = number of rings assumed within the  
object.

We note that radial symmetry is assumed and  
is now a function of  $r$  only. We also note that

$$S_{k,j} = 0, \text{ for } j < k \quad (6)$$

Since the  $k$ th ray does not intersect the  $j$ th ring if  $k$   
is less than  $j$ . Eqn.(5) can be rewritten in matrix  
notation, as

$$[d] = [S] [\mu] \quad (7)$$

where,

$[d] = (d_m \quad d_{m-1} \quad \dots \quad d_1)$ , the data vector,

$[\mu] = (\mu_m, \quad \mu_{m-1} \quad \dots \quad \mu_1)$ , the  $\mu$  vector ,

and

$$[S] = \begin{bmatrix} S_{m,m} & 0 & 0 \\ S_{m-1,m} & S_{m-1,m-1} & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ S_{1,m} & S_{2,m-1} & S_{1,1} \end{bmatrix}$$

is the chord-segment matrix which happens to be lower-triangular in this case.

By Eqn.(7), we get

$$[\mu] = [S]^{-1} [d] \quad (8)$$

Since the inverse of chord-segment matrix  $[S]$  is involved, this method is known as chord-segment-inversion technique. This results in  $\mu$  for various rings (or various intervals along the radius). A finer data vector will result in a better approximation of  $\mu$  along the radial line.

The expression for  $S_{k,j}$ 's is given by (See Appendix-A)

$$S_{k,j} = BC - B'C' \quad (\text{See Fig. 1})$$

$$= 2D \sqrt{(\sin \theta_{j+1})^2 - (\sin \theta_k)^2} - \sqrt{(\sin \theta_j)^2 - (\sin \theta_k)^2}$$

(9)

## 2.2 CHORD-SEGMENT-INVERSION (CSI) METHOD

The present chord-segment-inversion technique is relatively simple and faster than the general tomographic methods.

A FORTRAN program for the CSI algorithm has been written and implemented with the flexibility to change various geometrical variables, step size, error of integration, number of rings (See Appendix I) etc.

Now, for simulation studies, we want to obtain the data vector,  $[d]$ , from Eqn.(4). Since we have assumed that  $\mu(r, \theta)$  is radially symmetric function, i.e.

$$\mu(r, \theta) = \mu(r) \quad (10)$$

so Eqn.(4) can be rewritten as

$$d_{\theta} = \int_{C_{\theta}} \mu(r) \, ds. \quad (11)$$

For simplicity we replace the variable  $s$  by  $x$  and take the origin for  $x$  at the mid point of the chord. Thus,

$$d_{\theta} = \int_{C_{\theta}} \mu(r) \, dx \quad (12)$$

If for a chord at an angle  $\theta$ ,  $x_1$  is the lower limit and  $x_2$  is the upper limit for the variable  $x$ , then Eqn.(12) can be written as

---

$$d_{\theta} = \int_{x_1}^{x_2} \mu(r) dx \quad . \quad (13)$$

By the geometry (Fig.1) and Fig.A1 (Appendix-A)

$$r = \sqrt{(D \sin \theta)^2 + x^2} \quad (14)$$

Eqn.(13) reduces to

$$d_{\theta} = \int_{x_1}^{x_2} \mu(x) dx \quad (15)$$

The steps for the reconstruction of the density distribution are as follows:

- (1) Read data [d] in form of a column vector for all rays in the fan-beam.
- (2) Compute elements of the lower triangular [S] matrix using Eqn.(9) .
- (3) Compute  $\mu$ -values along the radial segments using Eqn.(8) .
- (4) Calibrate  $\mu$ -values to the density values.

For the Back substitution, averaging, plexi glass contribution and data see Appendices B,C,D and E .

We note that for simulation studies the data vector [d] will have to be generated by Eqn.(15).

### CHAPTER-3

#### VALIDATION AGAINST SIMULATED DATA AND RESULTS FOR BUBBLY AIR-WATER FLOWS

##### 3.1 VALIDATION AGAINST SIMULATED DATA

In this chapter, we discuss the results for the simulated data. We assume the radius of the pipe to be one unit and the distance of the source from the centre of the pipe to be two units. The pipe is further divided into twelve annular rings.

The CSI algorithm has been tested on the following assumed symmetric distributions:

$$\begin{aligned}\mu(r) &= 1.0 \\ \mu'(r) &= r \\ \mu(r) &= \exp(r) \\ \mu(r) &= \exp(-r) .\end{aligned}$$

For above mentioned test functions the errors in reconstruction  $\times 10^{-6}$ , 0.0088, 0.03 and 0.005 respectively. are  
Figure 2 shows the results of reconstruction along with the actual function distributions. For a listing of output see Appendix F .

The CSI algorithm has also been tested for annular flows (See Fig. 3), having the following two distributions:



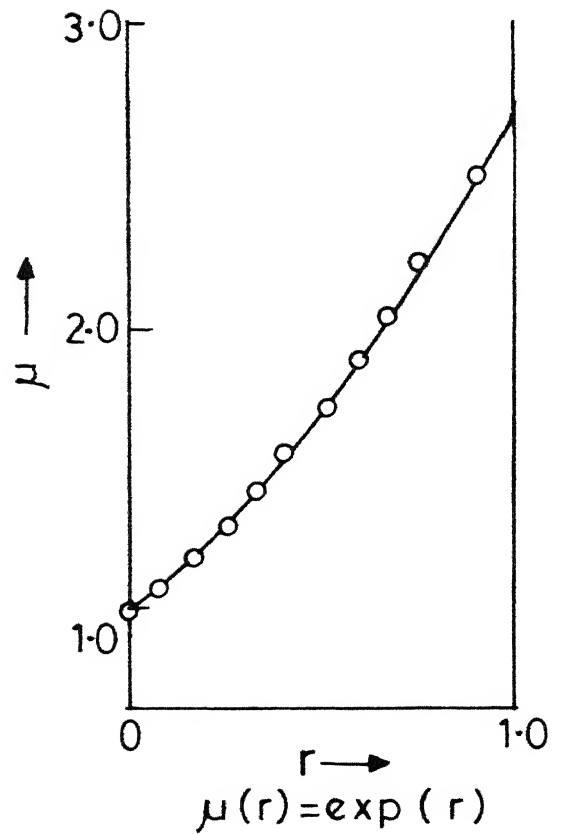
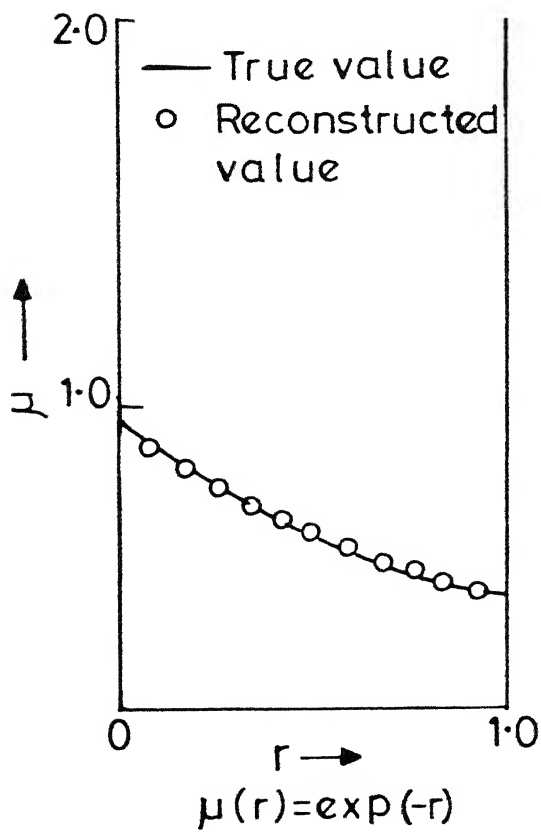
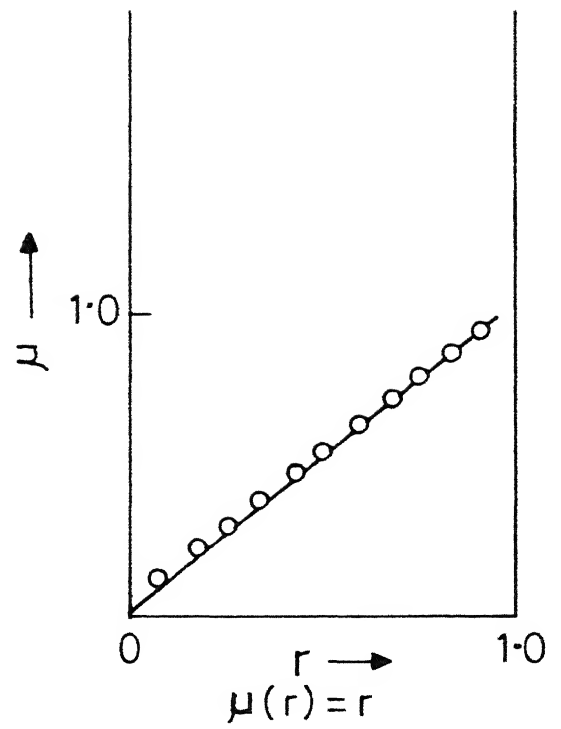
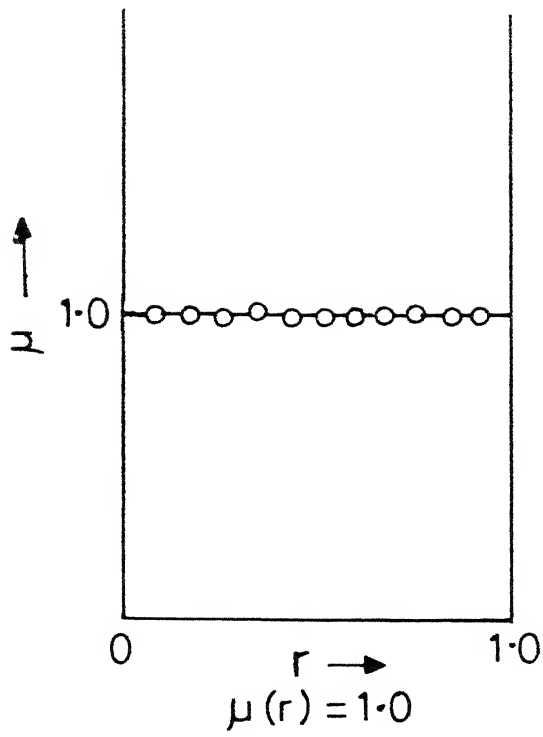


Fig.2 Reconstructed results for test-functions

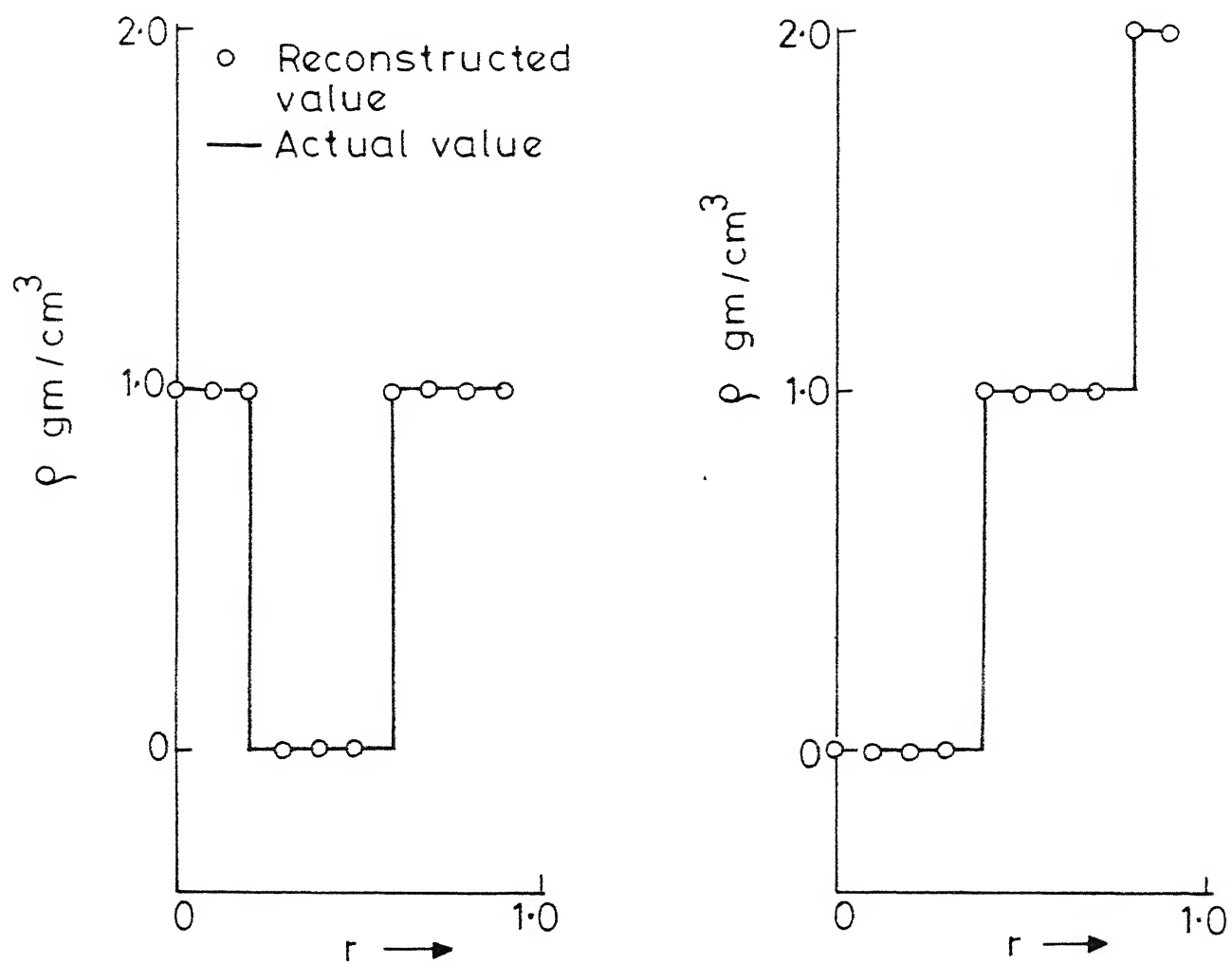


Fig.3 Results for simulated annular flow

$$\mu(r) = \begin{cases} 1.0, & 0 \leq r \leq 0.2 \\ 0.0, & 0.2 < r \leq 0.6 \\ 1.0, & 0.6 < r \leq 1.0 \end{cases}$$

$$\mu(r) = \begin{cases} 0.0, & 0 \leq r \leq 0.4 \\ 1.0, & 0.4 < r \leq 0.8 \\ 2.0, & 0.8 < r \leq 1.0 \end{cases}.$$

The reconstruction errors are almost negligible (See Appendix G).

The above mentioned-distributions represent (in a calibrated sense) the various types of density/void fraction distribution encountered in radially symmetric-bubbly and annular flows.

The reconstruction  $\mu$ -values matched the assumed-values very well for the simulated object of unit radius.

### 3.2 RESULTS FOR BUBBLY AIR-WATER FLOWS

Now here we will discuss the results for bubbly air-water flows. The data is taken from the study of Ref.[3,6]. Five different data-sets for four different cases of density (or void fraction) were processed by the CSI algorithm. The algorithm output,  $\langle \text{CTN} \rangle$ , had to be calibrated to obtain the density *value*. For this purpose, the previous work, included projection data for a few known cases of average density. Figure 4 (and Table F1, Appendix F)

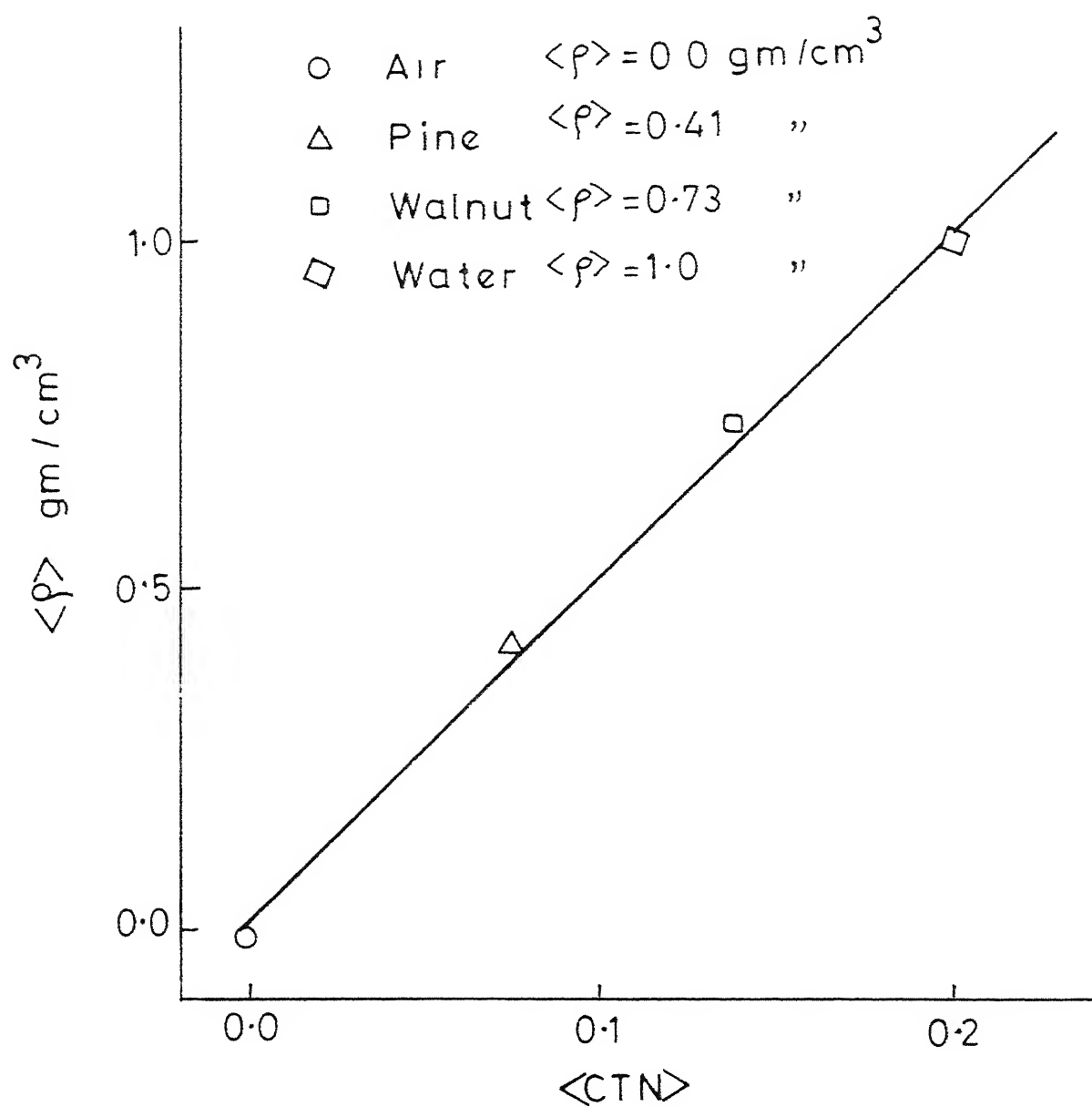


Fig.4 Calibration curve

shows the calibration chart resulting from the processing of the projection data for four known cases of densities. The output, CTN, was corrected to the base of air,  $CTN = 0.0$ , to eliminate the effect of plexiglass. (Details in Appendix D).

The four points are joined by a straight line such that the line represents the best fitting line for these four points. This figure is now the calibration curve because now by knowing the  $\langle CTN \rangle$  we can obtain the corresponding density value .

Similarly the data for all scans (See Appendix E) has been processed and we get the output (See Appendix H) in such a format that it gives the value of CTN for corresponding angle/radius. The value of  $\rho$  (obtained after calibration) for each radius has been plotted to show the reconstructed density and hence reconstructed profiles for various void fractions (See Figs 5-9).

In Figure 10 the comparison of reconstructed densities with actual densities has been shown by an alternative method [3.6] in which X-axis is the actual density and Y-axis is reconstructed density. A line such that  $\langle \rho \rangle = \langle \rho_{CT} \rangle$  has been drawn and various densities obtained from calibration method are located to show the deviation of the reconstructed densities from the actual densities. Appendix F summarises the  $\langle CTN \rangle$  and  $\langle \rho \rangle$  results for all scans in a tabular form.

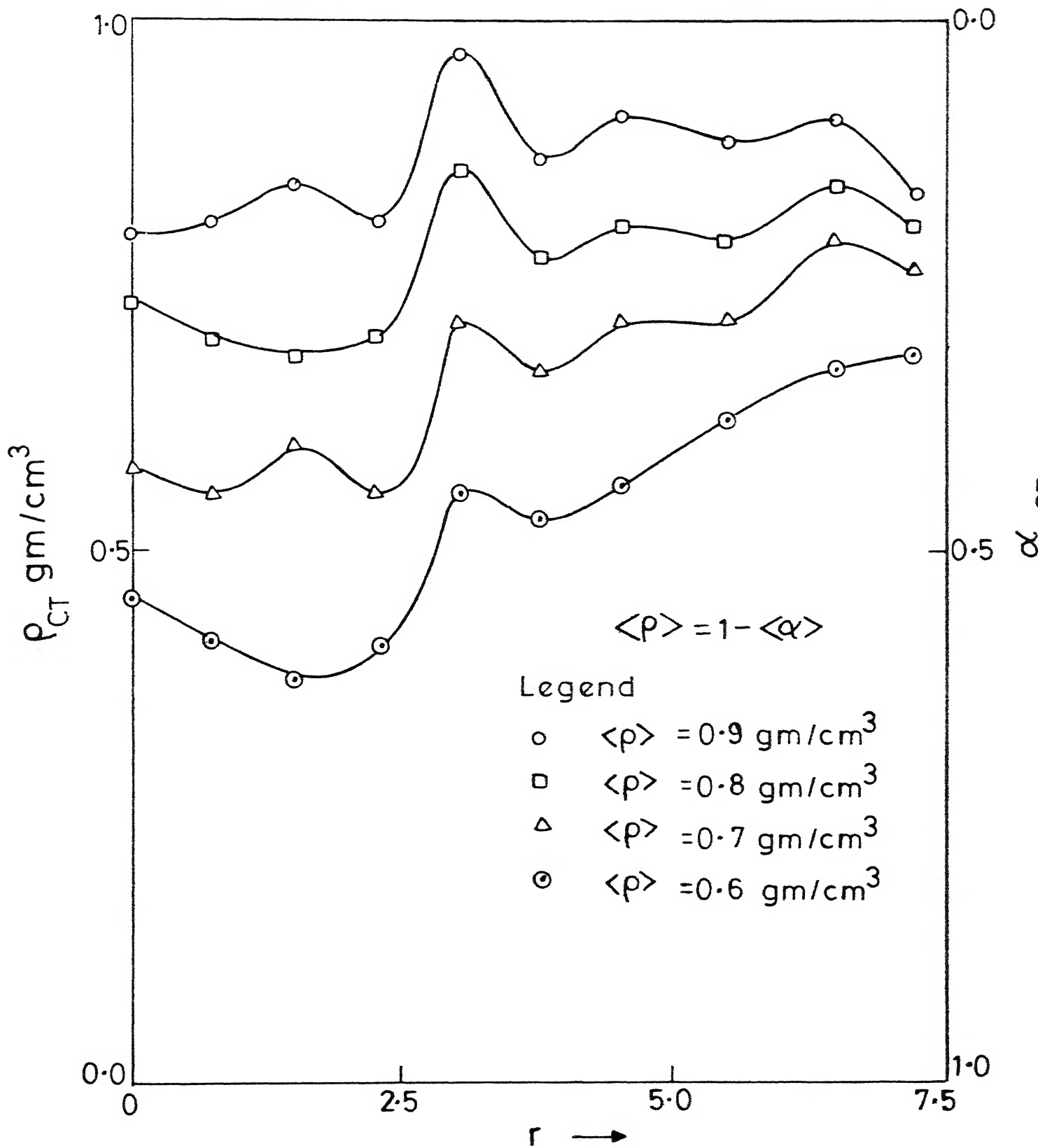


Fig.5 Reconstucted density profile for various density (average)cases for scan 1

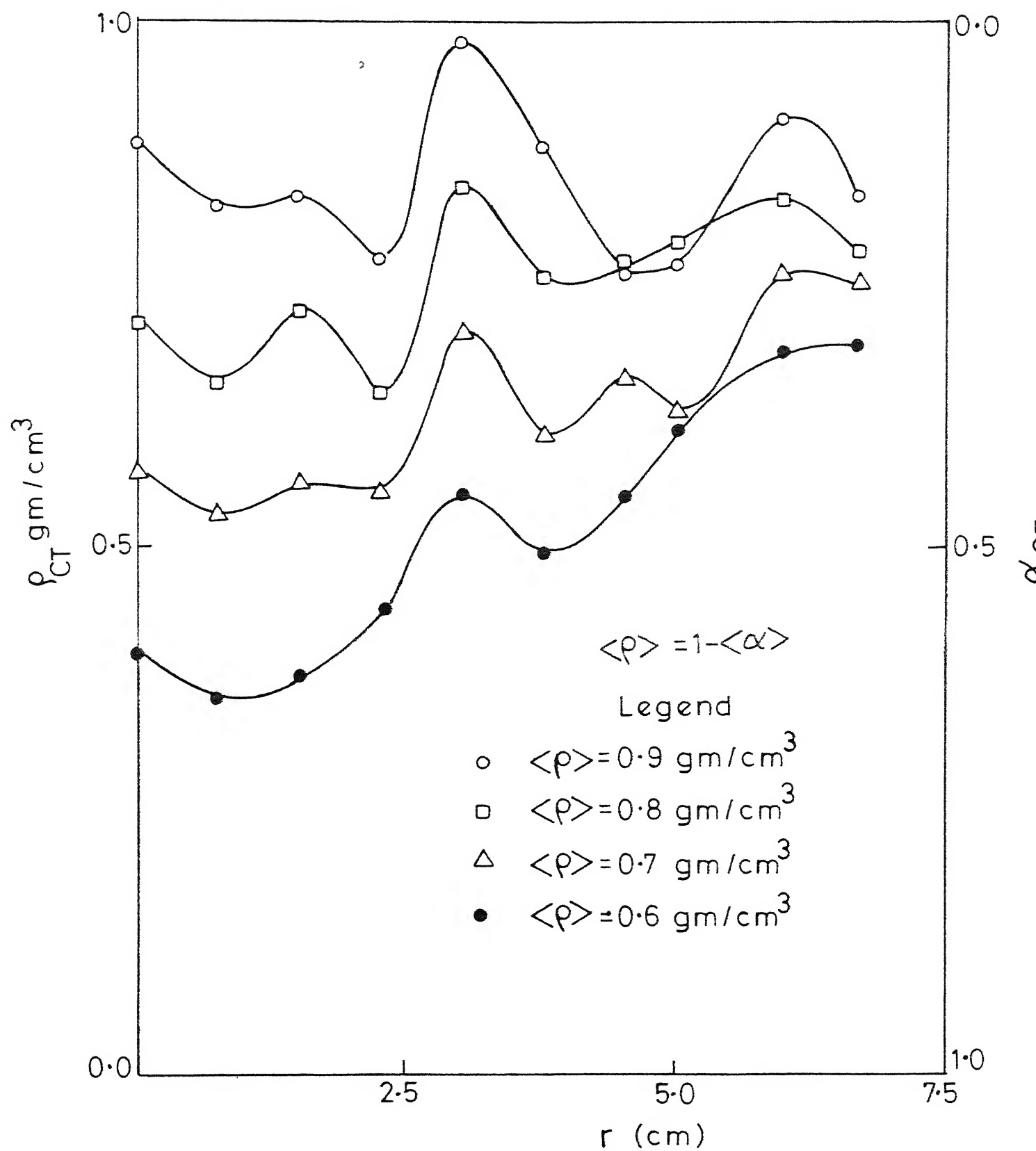


Fig.6 Density profile (radial) for scan 2

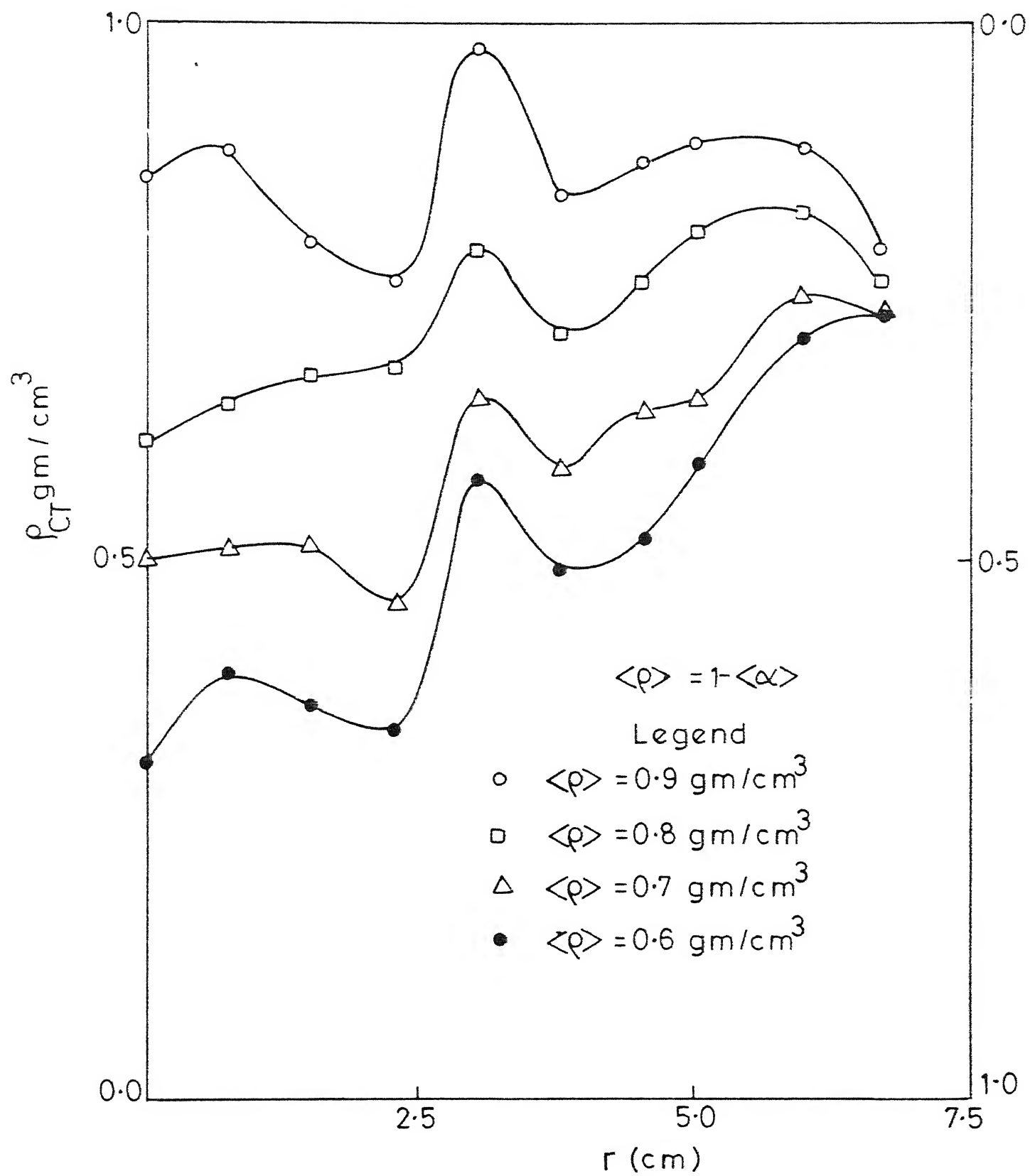


Fig.7 Density profile (radial) for scan 3



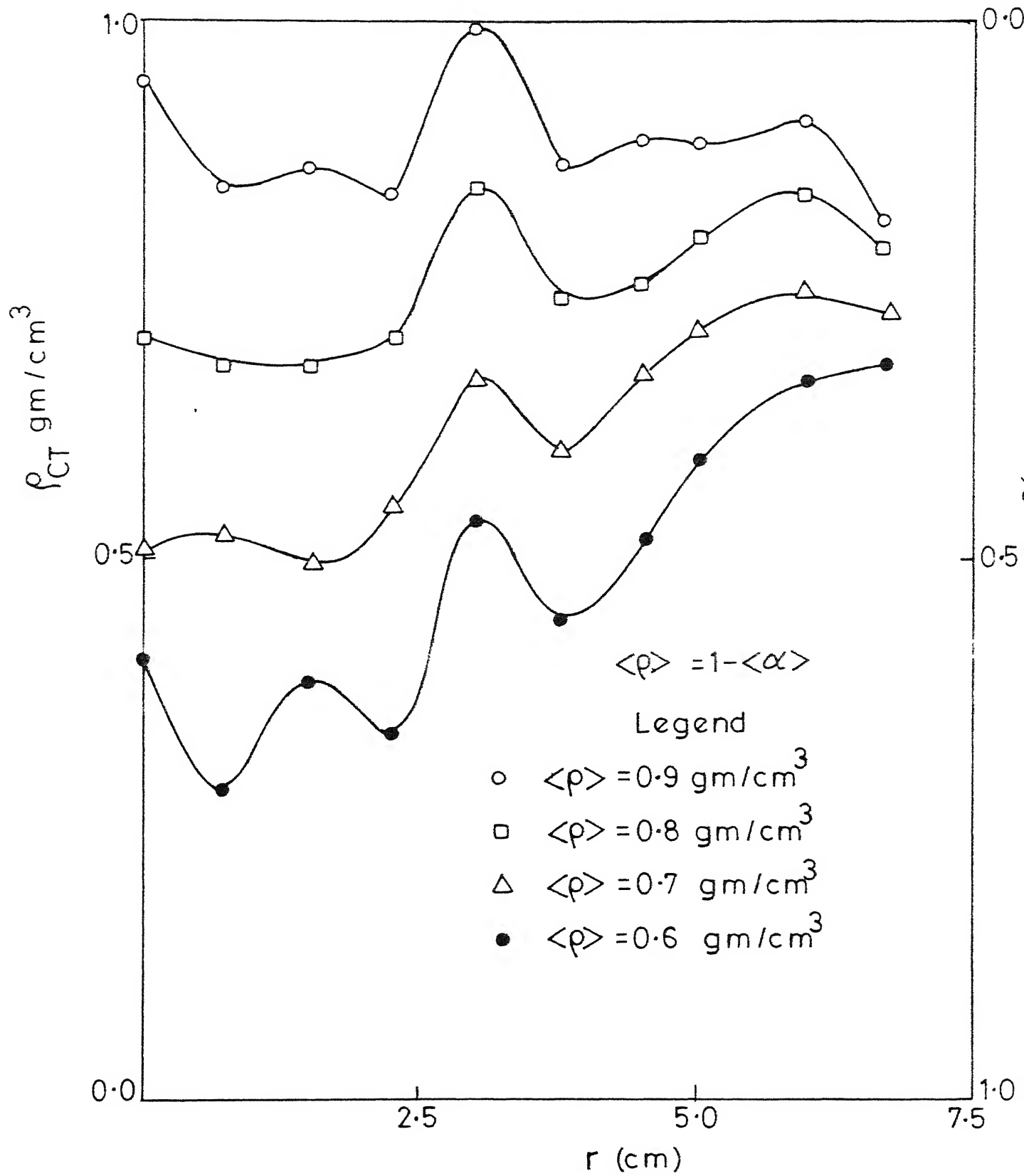


Fig.8 Density profile (radial) for scan 4

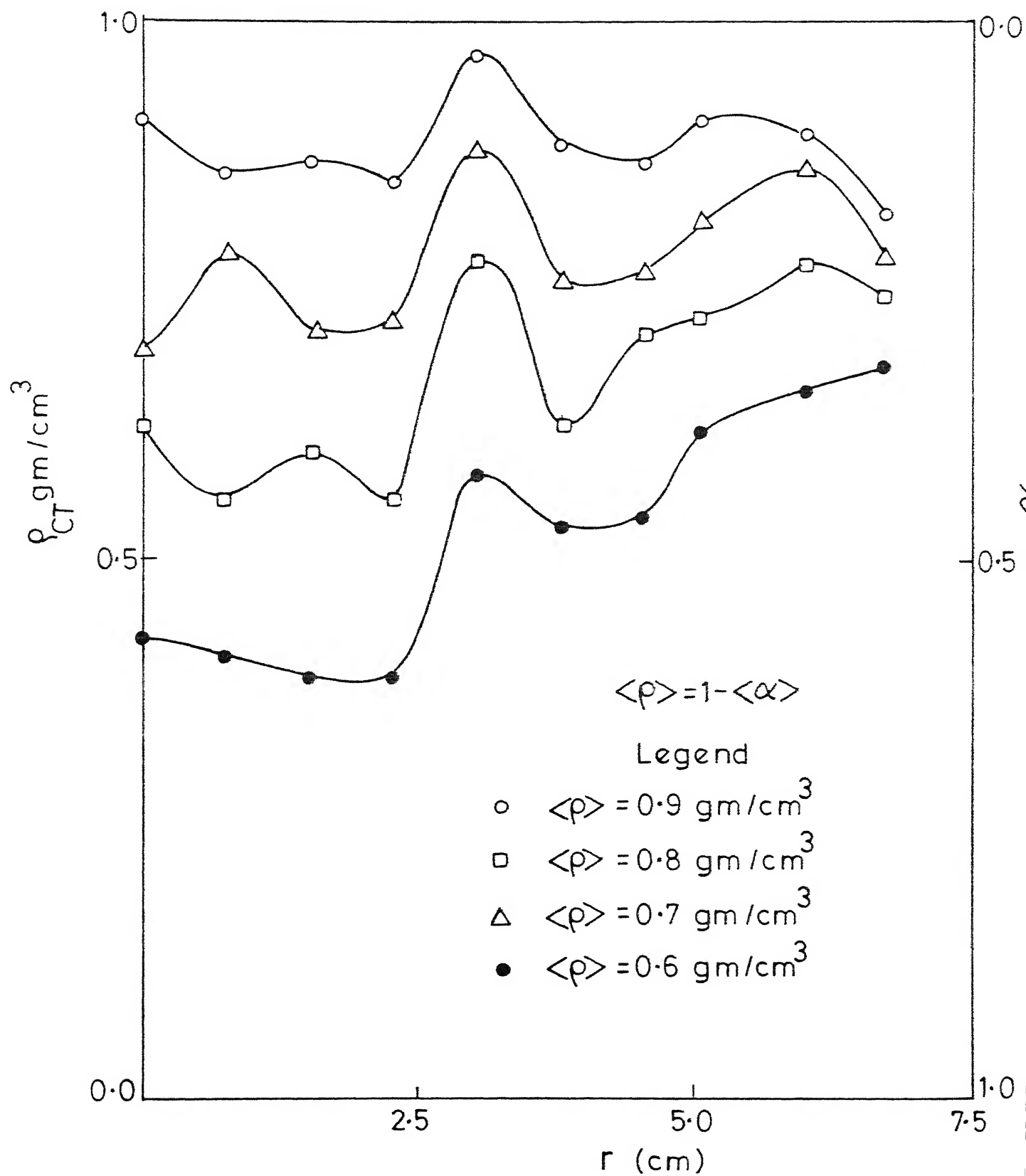


Fig.9 Density profile (radial) for scan 5

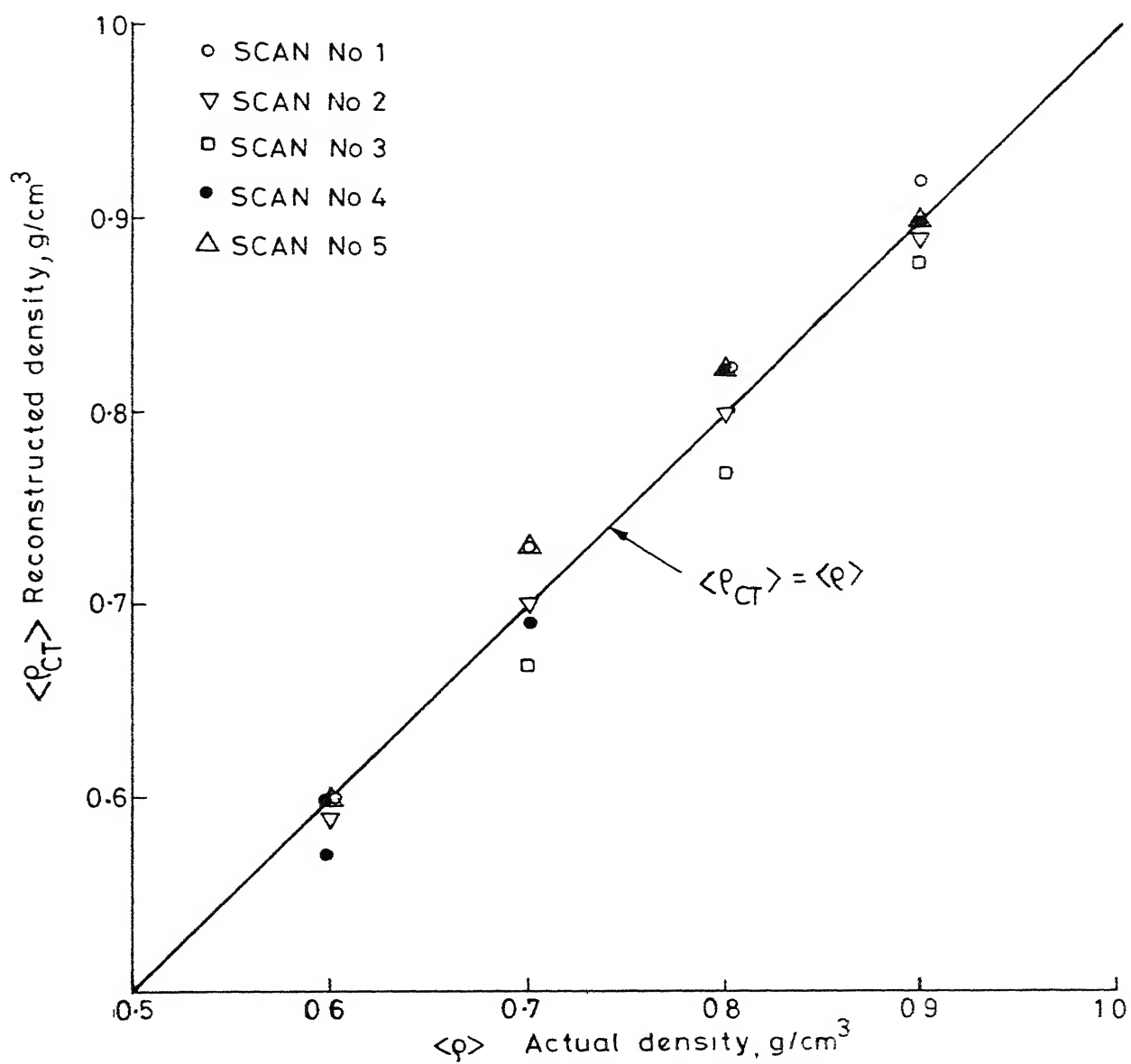


Fig.10 Comparison of reconstructed density with an alternate method

The maximum error obtained in reconstruction is  $\pm 0.03 \text{ gm/cm}^3$ . In Fig.11 the variation of error with density has been shown.

The maximum relative errors are + 4% and -5% respectively. In Fig.12 the variation of relative error with density has been shown.

We note that there are some statistical fluctuations in the count-rate recorded by the detector. This uncertainty leads to an erroneous reconstruction. Such a discrepancy appears to be quite obvious for constant density cases of air, pine, walnut and water (See Fig. 13 ). However assuming Poisson distribution and applying  $\pm 1 \sigma$  and  $\pm 3 \sigma$  corrections (where  $1 \sigma$  implies one standard deviation of the count rate,  $N$ ), the ripple appearing in Figure 13 is smoothed out as is evident from Figures 14 and 15. This exercise leads to a  $\rho_{CT}$  "band" for the air-water flow data. Since the point density values were not available to make any meaningful comparison, the air-water cases have not been presented.

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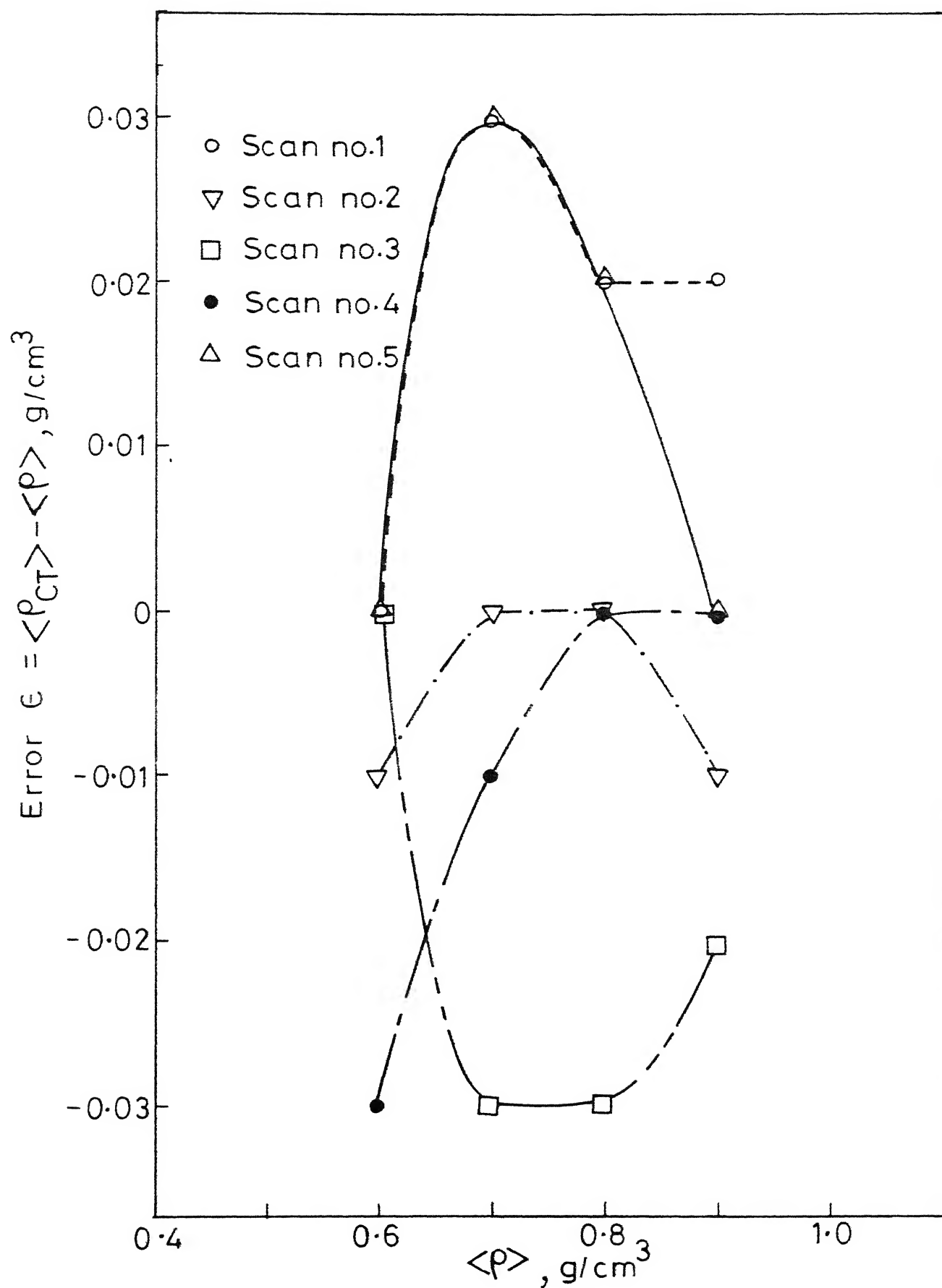


Fig.11 Error in density measurement for all scans

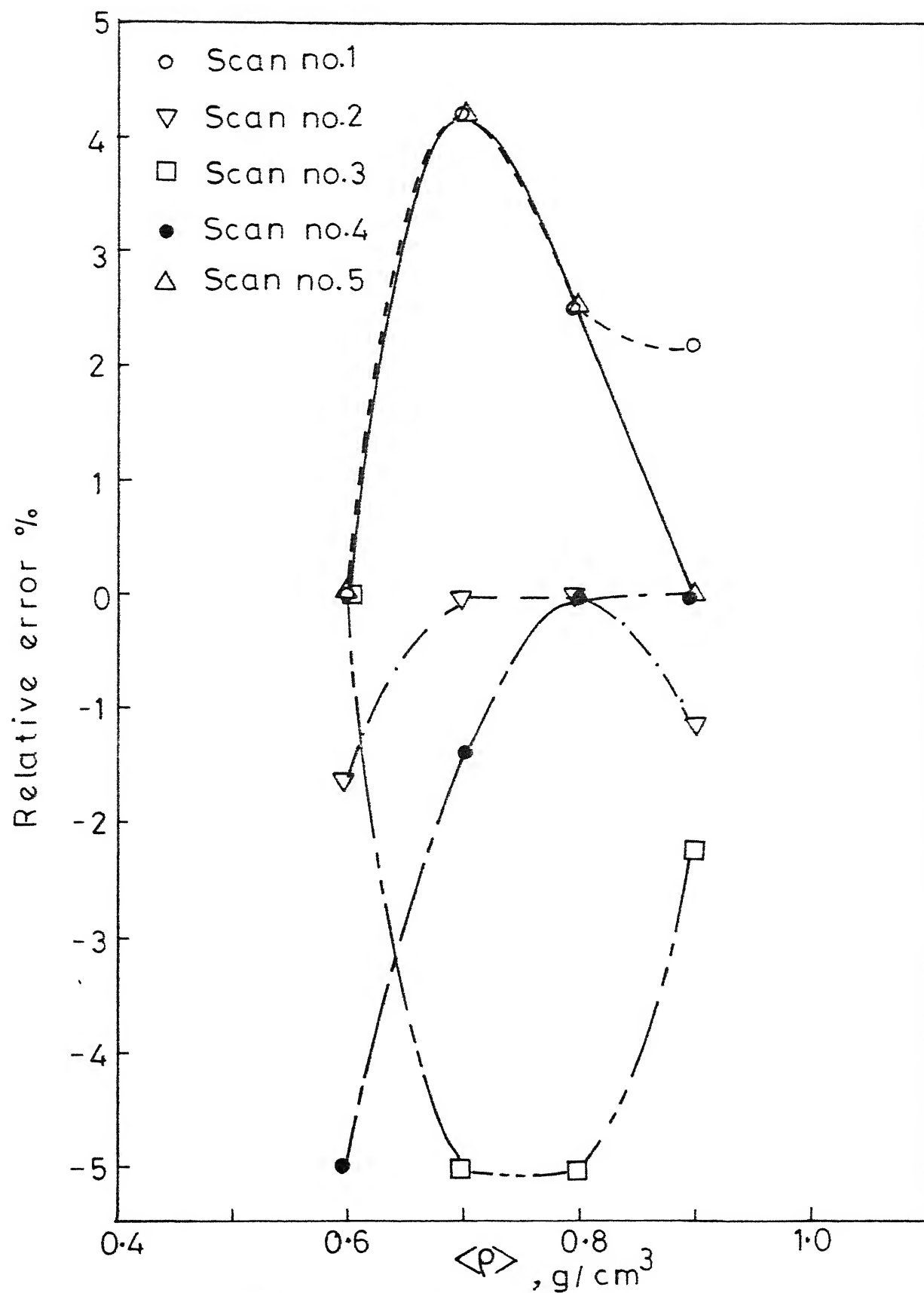


Fig.12 Relative error for all scans

○ Water	$\langle \rho \rangle = 1.0 \text{ g/cm}^3$
△ Walnut	$\langle \rho \rangle = 0.73 \text{ ''}$
× Pine	$\langle \rho \rangle = 0.41 \text{ ''}$
□ Air	$\langle \rho \rangle = 0.0 \text{ ''}$

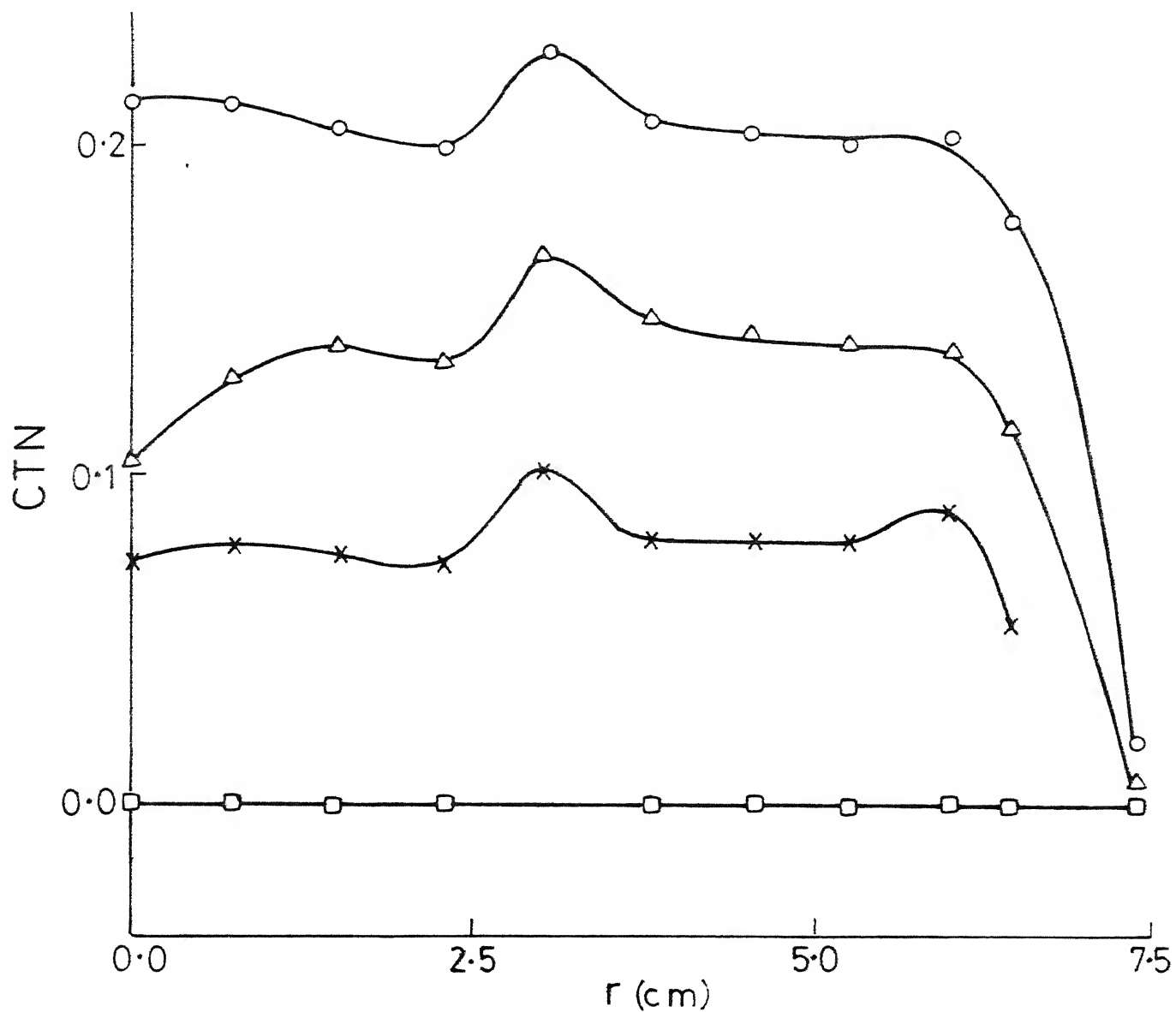
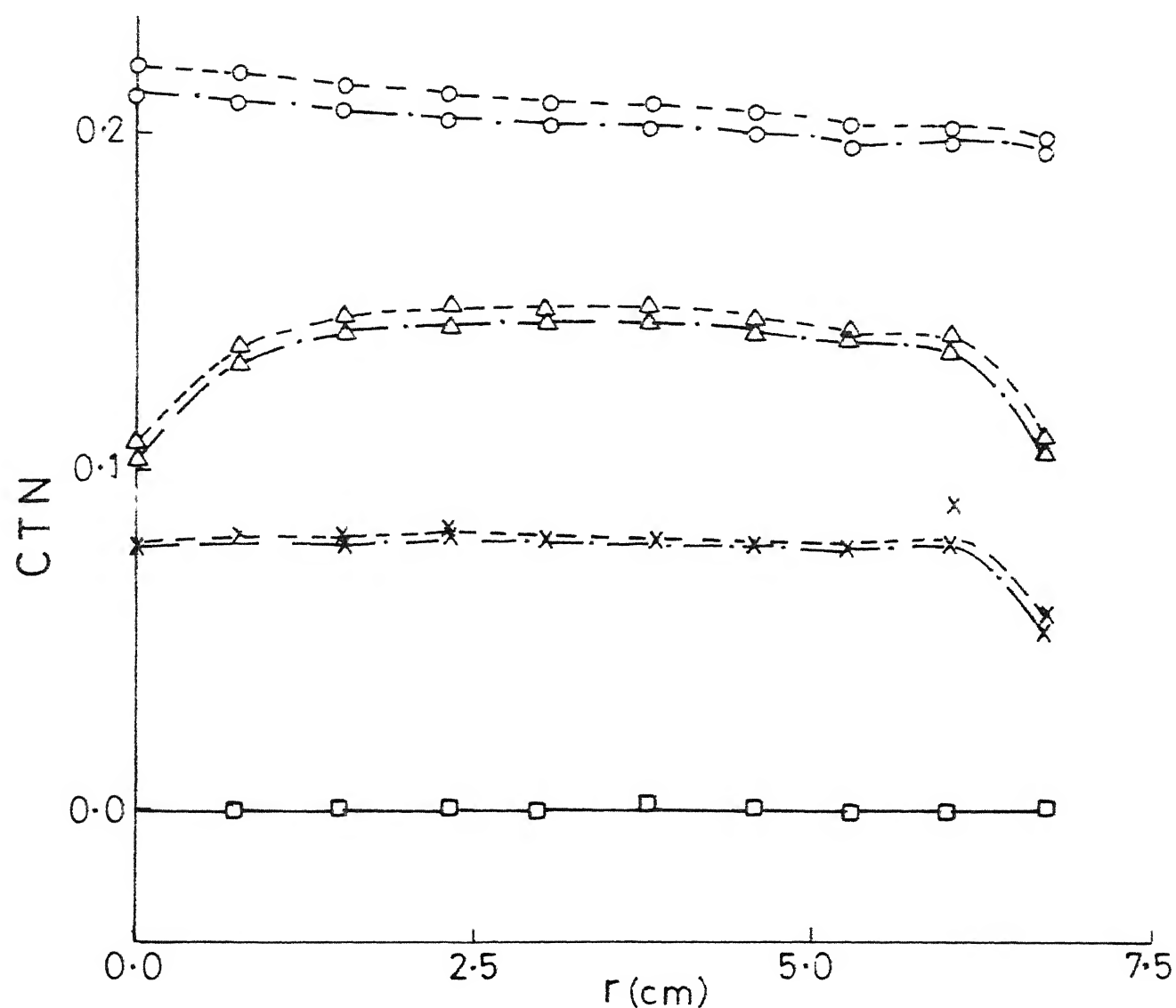


Fig.13 Calibration cases with mean value of counts

## POISSON STATISTICS

N: Mean value of counts

○ Water  $\langle \rho \rangle = 1.0 \text{ g/cm}^3$ △ Walnut  $\langle \rho \rangle = 0.73$  "x Pine  $\langle \rho \rangle = 0.41$  "□ Air  $\langle \rho \rangle = 0.0$  "---  $N_1 = N - \sqrt{N}$ -.-  $N_2 = N + \sqrt{N}$ Fig.14 Calibration cases with  $1-\sigma$  band



○ Water  $\langle \rho \rangle = 1.0 \text{ g/cm}^3$

△ Walnut  $\langle \rho \rangle = 0.73 \text{ "}$

x Pine  $\langle \rho \rangle = 0.41 \text{ "}$

□ Air  $\langle \rho \rangle = 0.0 \text{ "}$

POISSON STATISTICS

N: Mean value of count

---  $N_1 = N - 3\sqrt{N}$

-.-  $N_2 = N + 3\sqrt{N}$

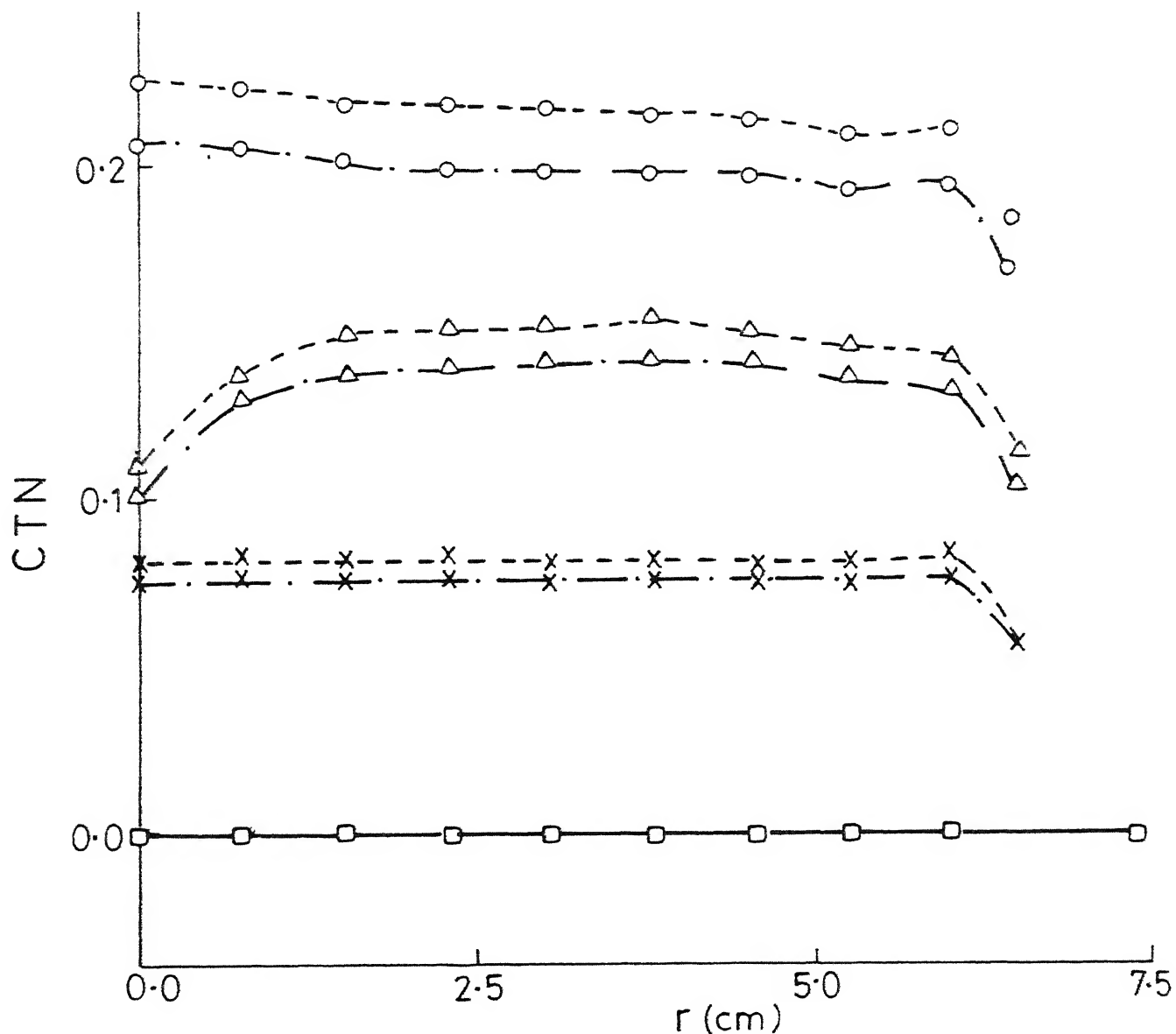


Fig.15 Calibration cases with 3- $\sigma$  band

## CHAPTER-4

### DISCUSSION

The proposed chord-segment inversion (CSI) technique has been demonstrated to be applicable to radially symmetric two-phase flows. The results are comparable to the results obtained by more general methods of tomographic reconstruction [2,3,6,7,8].

However, taking the advantage of the radial symmetry in the theoretical formulation itself, the CSI method consumes less Central Processing Unit (CPU) time and it is simple to use.

Comparable results have been obtained by CSI technique for density measurement in bubbly air-water flow for cases  $0.6 \leq \rho \leq 1.0$ . The maximum deviation of  $\rho_{CT}$  is about  $\pm 0.03\text{g/cm}^3$  which is approximately  $\pm 5\%$ . The maximum relative errors are +4% and -5% respectively. The error in the point density values could not be estimated as no alternate method was available in the study of Ref.[3,6]. But the test-function results indicate that for "pure" data the reconstruction is exact. The results of CSI method are also comparable to "radial polynomial" method already suggested and tested [9] for radially symmetric flow distributions.

---

APPENDIX-A

Suppose we have a pipe of radius  $R$  and a gamma ray source  $S$  at a distance  $D$  from the centre of the pipe. Consider now  $j$ th and  $(j+1)$ th annular rings and five chords  $C_{\theta_0}$ ,  $C_{\theta_k}$ ,  $C_{\theta_j}$ ,  $C_{\theta_{j+1}}$  and  $C_{\theta_{\max}}$  such that they make angles  $0$ ,  $\theta_k$ ,  $\theta_j$ ,  $\theta_{j+1}$ ,  $\theta_{\max}$  as shown in Fig.1.

The angle corresponding to chord  $C_{\theta_{\max}}$  is the maximum angle  $\theta_{\max}$  and is given by

$$\sin(\theta_{\max}) = R/D \quad (A-1)$$

$$\theta_{\max} = \sin^{-1} (R/D) \quad (A-2)$$

Note that any ray (chord)  $C_{\theta_n}$  is the tangent to the  $(n-1)^{\text{th}}$  annular ring.

Now we want to calculate the length of the segment of the  $k$ th ray falling in the  $j$ th annular ring that is  $S_{k,j}$ .

From Fig. 1 this length is the hatched line

$$S_{k,j} = BC - B'C' \quad (A-3)$$

Now see the detailed geometry of Fig. 1 in Fig.A1,

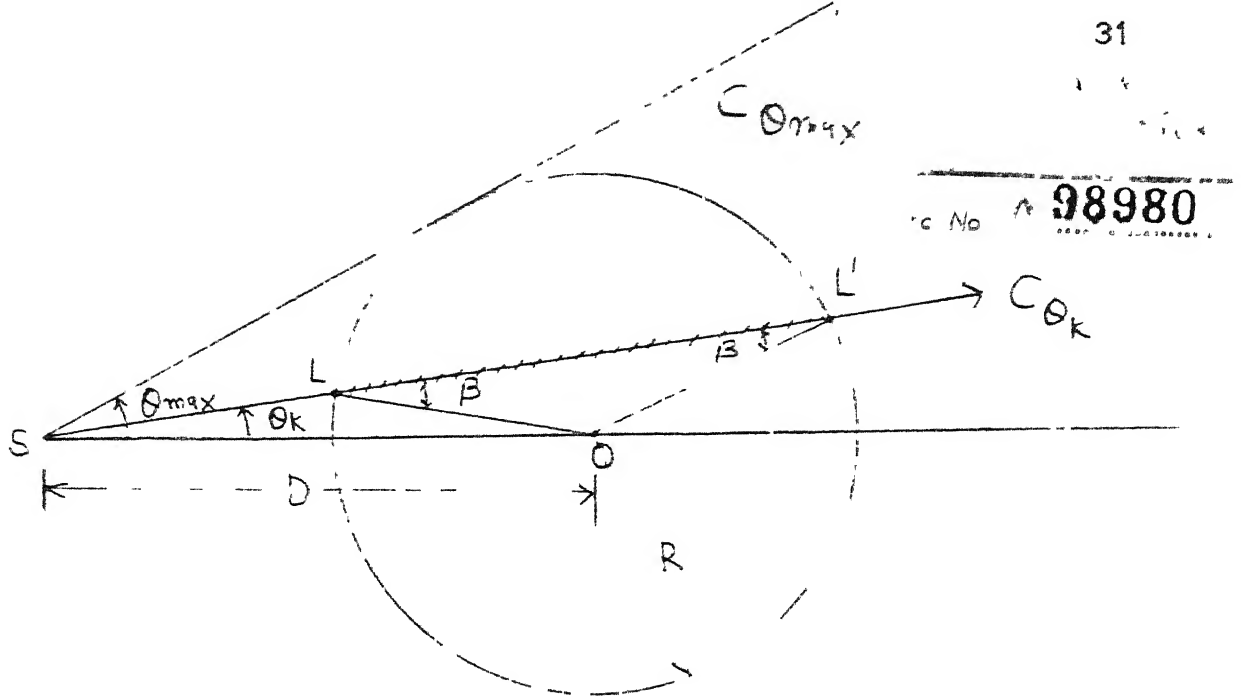


Figure A1 : Fan Beam Geometry Details

We are interested in calculating the length  $LL'$  in the  $C_{\theta_k}$  chord such that  $C_{\theta_{\max}}$  is the tangential chord.

From the triangle  $SOL'$

$$\frac{D}{\sin(\beta)} = \frac{R}{\sin(\theta_k)} \quad (A-4)$$

$$\beta = \sin^{-1} \left( \frac{D \sin(\theta_k)}{R} \right) \quad (A-5)$$

But from Eqn.(A-1)

$$R = D \sin(\theta_{\max}) \quad (A-6)$$

So from Eqn.(A-5)

$$\beta = \sin^{-1} \left( \frac{D \sin(\theta_k)}{D \sin(\theta_{\max})} \right) \quad (A-7)$$

$$= \sin^{-1} \left( \frac{\sin(\theta_k)}{\sin(\theta_{\max})} \right) \quad (\text{A-8})$$

The length  $LL'$

$$LL' = 2R \cos(\beta) \quad (\text{A-9})$$

$$= 2D \sin(\theta_{\max}) \cos\left(\sin^{-1}\left(\frac{\sin(\theta_k)}{\sin(\theta_{\max})}\right)\right) \quad (\text{A-10})$$

$$= 2D \sin(\theta_{\max}) \sqrt{1 - \frac{\sin^2 \theta_k}{\sin^2 \theta_{\max}}} \quad (\text{A-11})$$

$$= 2D \sqrt{\sin^2 \theta_{\max} - \sin^2 \theta_k} \quad (\text{A-12})$$

By this token we have in Fig. 1.

$$BC = 2D \sqrt{\sin^2 \theta_{j+1} - \sin^2 \theta_k} \quad (\text{A-13})$$

$$B'C' = 2D \sqrt{\sin^2 \theta_j - \sin^2 \theta_k} \quad (\text{A-14})$$

hence the length

$$\begin{aligned} S_{k,j} &= BC - B'C' \\ &= 2D \left[ \sqrt{\sin^2 \theta_{j+1} - \sin^2 \theta_k} - \sqrt{\sin^2 \theta_j - \sin^2 \theta_k} \right] \end{aligned} \quad (\text{A-15})$$

APPENDIX-B

The Back Substitution is as follows:

Recalling equation (7)

$$[d] = [S] [\mu] \quad (B-1)$$

The expanded form of this equation is

$$\begin{Bmatrix} d_m \\ d_{m-1} \\ \vdots \\ d_1 \end{Bmatrix} = \begin{bmatrix} S_{m,m} & & 0 \\ S_{m-1,m} & S_{m-1,m-1} & \\ \vdots & \vdots & \vdots \\ S_{1,m} & S_{2,m-1} & S_{1,1} \end{bmatrix} \begin{bmatrix} \mu_m \\ \mu_{m-1} \\ \vdots \\ \mu_1 \end{bmatrix} \quad (B-2)$$

Then the algebraic equations will be

$$d_m = S_{m,m} \mu_m \quad (B-3)$$

$$d_{m-1} = S_{m-1,m} \mu_m + S_{m-1,m-1} \mu_{m-1} \quad (B-4)$$

$$\begin{aligned} d_{m-2} &= S_{m-2,m} \mu_m + S_{m-2,m-1} \mu_{m-1} + S_{m-2,m-2} \mu_{m-2} \\ &\vdots \\ &\vdots \\ &\vdots \end{aligned} \quad (B-5)$$

$$d_1 = S_{1,m} \mu_m + S_{2,m-1} \mu_{m-1} + \dots + S_{1,1} \mu_1 \quad (B-6)$$

From Equation (B-3) we have,

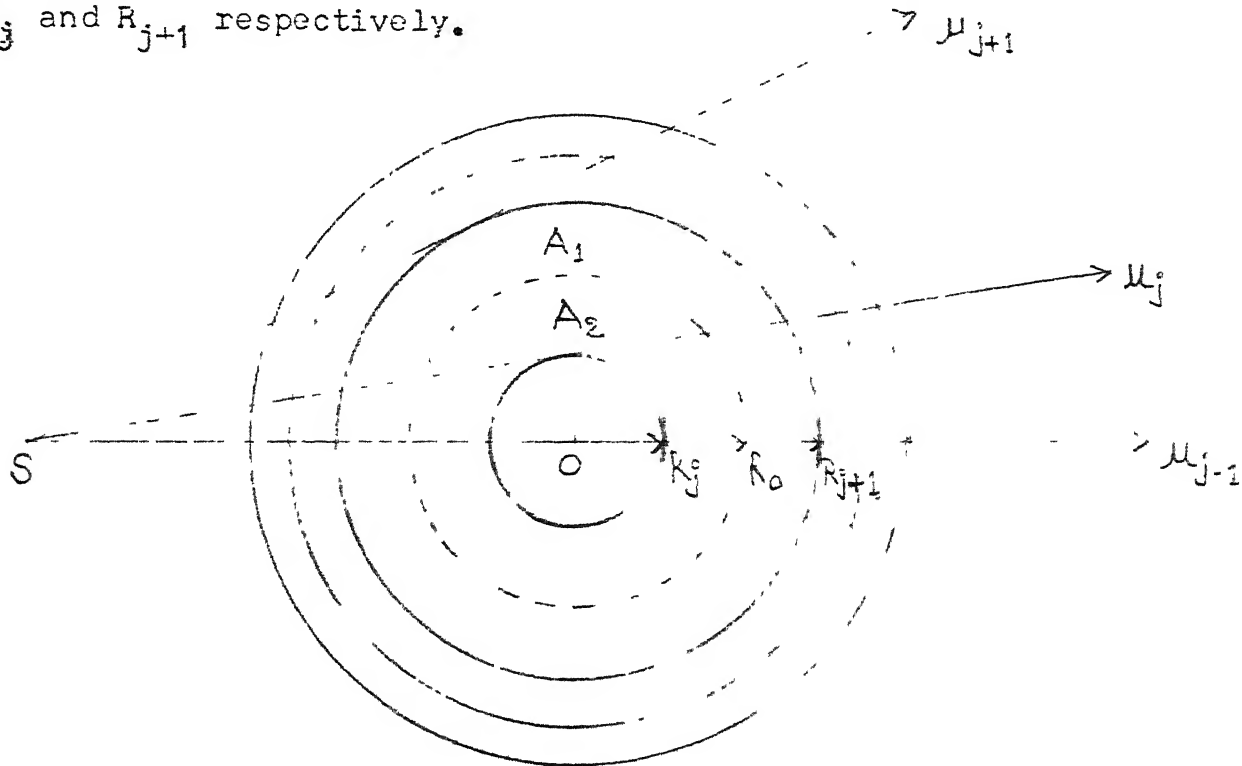
$$\mu_m = \frac{d_m}{S_{m,m}}$$

Substituting this value of  $\mu_m$  in (B-4) we obtain the value of  $\mu_{m-1}$  and then using the value of  $\mu_{m-1}$  in next equation to get  $\mu_{m-2}$  and so on.

So by back substitution we get the values of all  $\mu$ 's.

## APPENDIX C

For the simulated data study of radially symmetric distributions, the  $\mu$  has been reconstructed. In Fig. C1,  $\mu_{j-1}$ ,  $\mu_j$  and  $\mu_{j+1}$  are the reconstructed values of  $\mu$ 's corresponding to radius 0,  $R_j$  and  $R_{j+1}$  respectively. Similarly  $\bar{\mu}_{j-1}$ ,  $\bar{\mu}_j$  and  $\bar{\mu}_{j+1}$  are the actual  $\mu$ 's at radius 0,  $R_j$  and  $R_{j+1}$  respectively.



**Fig-C1 AVERAGE-VALUES**

Now take the average values of  $\bar{\mu}$ 's  $\langle \bar{\mu}_{j-1} \rangle$ ,  $\langle \bar{\mu}_j \rangle$  and  $\langle \bar{\mu}_{j+1} \rangle$  such that they are the values of  $\bar{\mu}$ 's in first, second and third annular rings. Now  $\langle \bar{\mu}_j \rangle$  is the value of  $\bar{\mu}$  at radius  $R_0$  as shown in Fig. C1.



If area between  $R_{j+1}$  and  $R_o$  is  $A_1$  then

$$A_1 = \pi R_{j+1}^2 - \pi R_o^2$$

Similarly if area between  $R_o$  and  $R_j$  is  $A_2$  then

$$A_2 = \pi R_o^2 - \pi R_j^2$$

Taking  $A_1 = A_2$

$$R_o = \sqrt{\frac{R_j^2 + R_{j+1}^2}{2}}$$

i.e. the average radius of the annular ring having  $R_j$  internal radius and  $R_{j+1}$  outer radius. This must be taken into account when calculating  $\mu$  at various radii.

## APPENDIX D

The data for all scans has been taken from [6] . There was plexi-glass around the pipe in the experiment of [6] for all scans. For air the CTN (absorption coefficient) must be zero but for air data set we do not get the CTNs (corresponding to different angles) equal to zero but some non-zero values, this is due to the plexi-glass whatever CTNs we get will be the CTNs for plexi-glass but not for air.

So this plexi-glass contribution must be taken into account in order to obtain the correct values of CTNs for all cases. For this we make air as the reference and all CTNs equal to zero by subtracting CTNs of plexi-glass. Similarly for any case we subtract the CTNs of plexi-glass to get the correct CTNs and hence correct  $\langle \text{CTNs} \rangle$ .

APPENDIX F

TABLE-F1

(Results for air-pine walnut and water)

Case	$\langle \rho \rangle$ , g/cm <sup>3</sup>	$\langle \text{CTN} \rangle$
Air	0.0	0.000
Pine	0.41	0.075
Walnut	0.732	0.137
Water	1.00	0.201

TABLE-F2

(Reconstructed Densities)

Scan No.	$\langle \rho \rangle$ gm/cm <sup>3</sup>	0.6	0.7	0.8	0.9
1		0.60	0.73	0.82	0.92
2		0.59	0.70	0.8	0.89
3		0.60	0.67	0.77	0.88
4		0.57	0.69	0.80	0.90
5		0.6	0.73	0.82	0.90

TABLE-F3

(Reconstructed <CTNs> )

Scan No.	$\langle \rho \rangle$ gm/cm <sup>3</sup>	0.6	0.7	0.8	0.9
1		0.116	0.141	0.159	0.175
2		0.114	0.136	0.155	0.173
3		0.111	0.129	0.152	0.171
4		0.110	0.134	0.155	0.176
5		0.115	0.141	0.159	0.176

Table F 1 Scan No. 1

## DEGREES

TYPE SCAN	-30	-27.5	-25	-22.5	-20	-17.5	-15	-12.5	-10	-7.5	-5	-2.5	0	2.5	5	7.5	10	12.5	15	17.5	20	22.5	25	27.5	30
AIR	2869	2063	2084	2315	2404	2447	2485	2490	2516	2528	2534	2532	2527	2522	2526	2513	2492	2471	2449	2398	2308	2149	1743	2834	2842
46% VOID	2875	2109	1573	1518	1483	1448	1435	1434	1443	1456	1465	1464	1470	1469	1463	1466	1456	1423	1452	1464	1482	1541	1703	2830	2832
40% VOID	2826	2100	1552	1469	1402	1380	1351	1359	1332	1345	1349	1348	1342	1348	1361	1345	1357	1342	1364	1384	1422	1497	1667	2835	2846
30% VOID	2857	2061	1517	1377	1278	1218	1176	1160	1146	1130	1133	1126	1136	1139	1141	1158	1171	1176	1209	1269	1314	1430	1646	2835	2837
20% VOID	2864	2074	1500	1331	1218	1133	1079	1055	1018	1010	1002	1001	999	1005	1018	1023	1048	1077	1125	1198	1261	1390	1630	2824	2819
10% VOID	2879	2035	1505	1308	1173	1082	1005	958	913	894	884	876	894	894	902	923	952	991	1041	1127	1218	1370	1632	2800	2815
WATER	2885	2062	1495	1266	1106	996	913	858	811	781	758	748	740	745	765	790	826	877	951	1044	1153	1359	1663	2850	2835
WALNUT ( $\rho = .732$ )	2853	2082	1742	1565	1449	1392	1335	1265	1206	1159	1141	1121	1126	1108	1113	1132	1161	1207	1282	1365	1458	1619	1705	2846	2846
PINE ( $\rho = .41$ )	2858	2060	1773	1863	1800	1754	1705	1678	1639	1616	1601	1599	1595	1594	1610	1621	1645	1669	1715	1750	1782	1856	1710	2834	2827









Table E.6 Scan No. 5,

[illegible]

RADIUS R= 1.00000000

DISTANCE OF SOURCE D= 2.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

ERROR OF INTEGRATION E= 0.00000010

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)=1.000

TH	DATA	r	F(r)	FC(r)	PRODUCT
0.00	2.00000000	0.05168713	1.00000000	1.00000010	0.02390942
2.50	1.99237480	0.13783159	1.00000000	0.99999997	0.07154523
5.00	1.98938110	0.22196040	1.00000000	0.99999997	0.41853865
7.50	1.93064930	0.30721581	1.00000000	1.00000000	0.15482811
10.00	1.87551090	0.39242782	1.00000000	1.00000000	0.20975314
12.50	1.80290380	0.47714443	1.00000000	0.99999999	0.25310177
15.00	1.71119940	0.56109051	1.00000000	1.00000000	0.29451405
17.50	1.59787870	0.64405242	1.00000000	0.99999999	0.33358492
20.00	1.45888850	0.72584350	1.00000000	0.99999999	0.37031630
22.50	1.28718850	0.80629127	1.00000000	0.99999999	0.40412931
25.00	1.06878470	0.88523215	1.00000000	0.99999999	0.43486669
27.50	0.76721019	0.95250900	1.00000000	1.00000000	0.45229401

AVERAGE DISTRIBUTION =1.00000000

RADIUS R= 1.00000000

DISTANCE OF SOURCE D= 2.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

ERROR OF INTEGRATION E= 0.00000010

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)=r

TH	DATA	r	F(r)	F2(r)	PRODUCT
0.00	1.00000000	0.05168713	0.05168713	0.06833501	0.00163385
2.50	1.02001120	0.13783159	0.13783159	0.13832951	0.00989597
5.00	1.05859700	0.22195040	0.22195040	0.22091244	0.02620875
7.50	1.10289510	0.30721581	0.30721581	0.30511155	0.05924095
10.00	1.14510510	0.39242782	0.39242782	0.38948959	0.08170054
12.50	1.17876480	0.47714443	0.47714443	0.47348329	0.11983043
15.00	1.19768690	0.55109051	0.55109051	0.55571619	0.15395065
17.50	1.19523980	0.64405242	0.64405242	0.63899919	0.21322455
20.00	1.16344980	0.72584350	0.72584350	0.71995514	0.25651482
22.50	1.09130230	0.80629127	0.80629127	0.79930034	0.32402070
25.00	0.95038473	0.88523215	0.88523215	0.87539731	0.33111501
27.50	0.72839410	0.95250900	0.95250900	0.94940629	0.43890513

AVERAGE DISTRIBUTION = .55094534

RADIUS R= 1.00000000

DISTANCE OF SOURCE D= 2.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

ERROR OF INTEGRATION E= 0.00000010

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION  $F(r) = \exp(-r)$

TH	DATA	r	F(r)	F2(r)	PRODUCT
0.00	1.25424110	0.05168713	0.94017599	0.93853305	0.92244093
2.50	1.23982890	0.13783159	0.87124540	0.87427597	0.95255120
5.00	1.18751500	0.22196040	0.80094708	0.80444957	0.99543881
7.50	1.11873130	0.30721581	0.73549186	0.73913804	0.92183073
10.00	1.03903740	0.39242782	0.67541510	0.67906952	0.94244375
12.50	0.95179631	0.47714443	0.62055289	0.62415412	0.95797447
15.00	0.85905208	0.55109051	0.57058649	0.57411547	0.95908503
17.50	0.76183482	0.64405242	0.52515993	0.52862952	0.91763957
20.00	0.66017219	0.72584350	0.48391622	0.48737548	0.93048303
22.50	0.55274329	0.80629127	0.44551099	0.45005972	0.93138633
25.00	0.43562403	0.88523215	0.41251837	0.41654015	0.93113945
27.50	0.29696561	0.95250900	0.38193341	0.38707202	0.91789412

AVERAGE DISTRIBUTION = .53177192

RADIUS R= 1.00000000

DISTANCE OF SOURCE D= 2.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

ERROR OF INTEGRATION E= 0.00000010

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION- $F(r)=EXP(r)$

TH	DATA	r	F(r)	F2(r)	PRODUCT
0.00	3.43656360	0.05168713	1.05362950	1.07844110	0.02578490
2.50	3.45351180	0.13783159	1.14778220	1.15483840	0.03262798
5.00	3.48265620	0.22196040	1.24852190	1.25294630	0.14354787
7.50	3.51017750	0.30721581	1.35953430	1.36192470	0.22448443
10.00	3.52634790	0.39242782	1.48057100	1.48089520	0.31063722
12.50	3.52151720	0.47714443	1.61446620	1.60980930	0.40744561
15.00	3.48462340	0.56109051	1.75258270	1.74379030	0.51504350
17.50	3.40176500	0.64405242	1.90418180	1.89794050	0.63331416
20.00	3.25398240	0.72584350	2.05647340	2.05722110	0.76182250
22.50	3.01271470	0.80629127	2.23958650	2.22525410	0.89969857
25.00	2.52793090	0.88523215	2.42354690	2.40380010	1.04533250
27.50	1.98312900	0.95250900	2.61325710	2.58485750	1.19495520

AVERAGE DISTRIBUTION =1.93937400

[illegible]

NUMBER OF ANNULUS RINGS N= 10

TABLE: DISTRIBUTION-F(c)=ANNULAR FLOW

TH	DATA	r	F(r)	F2(r)	ERROR
0.00	1.40000000	0.00000000	1.00000000	1.00000010	-0.00000005
2.87	1.37244430	0.10000000	1.00000000	0.99999997	0.00000003
5.74	1.27543450	0.20000000	1.00000000	0.99999999	0.00000001
8.63	0.86864796	0.30000000	0.00000000	0.00000007	-0.00000007
11.54	0.93860307	0.40000000	0.00000000	0.00000002	0.00000002
14.48	1.05872580	0.50000000	0.00000000	0.00000010	0.00000010
17.46	1.60000000	0.60000000	1.00000000	0.99999999	0.00000001
20.49	1.42828570	0.70000000	1.00000000	1.00000010	-0.00000007
23.58	1.29000000	0.80000000	1.00000000	1.00000000	-0.00000003
26.78	0.87177989	0.90000001	1.00000000	1.00000020	-0.00000010



RADIUS R= 1.00000000  
 DISTANCE OF SOURCE D= 2.00000000  
 STARTING ANGLE TH= 0.00000000  
 ERROR OF INTEGRATION E= 0.00000010  
 NUMBER OF ANNULUS RINGS N= 10

TABLE: DISTRIBUTION-F(r)=ANNULAR FLOW

TH	DATA	r	F(r)	F2(r)	ERROR
0.00	1.50000000	0.00000000	0.00000000	0.00000000	-0.00000000
2.87	1.51790220	0.10000000	0.00000000	0.00000022	0.00000022
5.74	1.67716990	0.20000000	0.00000000	0.00000010	0.00000010
8.63	1.80336690	0.30000000	0.00000000	0.00000000	0.00000000
11.54	2.28942000	0.40000000	1.00000000	1.00000020	-0.00000018
14.48	2.21510200	0.50000000	1.00000000	0.99999990	0.00000010
17.46	2.14169950	0.60000000	1.00000000	1.00000000	0.00000000
20.49	2.08197480	0.70000000	1.00000000	1.00000010	-0.00000010
23.58	2.40000000	0.80000000	2.00000000	2.00000010	-0.00000010
26.74	1.74355980	0.90000001	2.00000000	2.00000030	-0.00000030

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION- $F(r) = .000$  (PLEXI GLASS CTN)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.83478810	0.00000000	0.00000000	0.70291688	0.20587758
2.50	7.83280750	0.30533571	0.00000000	0.70404682	0.61705618
5.00	7.83439230	0.61009019	0.00000000	0.711773890	1.04310670
7.50	7.82923260	0.91368333	0.00000000	0.73290801	1.47084700
10.00	7.82084090	1.21553720	0.00000000	0.75374539	1.93652250
12.50	7.81237820	1.51507730	0.00000000	0.78445349	2.43221620
15.00	7.80343510	1.811173330	0.00000000	0.82922001	2.99155760
17.50	7.78239040	2.10494060	0.00000000	0.88705613	3.62595550
20.00	7.74413660	2.39414100	0.00000000	0.96427954	4.37433370
22.50	7.67275790	2.67878400	0.00000000	1.06743890	5.23142060
25.00	7.46336310	2.95832780	0.00000000	1.11605990	5.94543460

AVERAGE = 0.87259229



DISTANCE OF SOURCE D= 7.40000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)= .0000(AIP), [SCAN-1]

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.83478810	0.00000000	0.00000000	0.00000000	0.00000000
2.50	7.83280750	0.30533571	0.00000000	0.00000000	0.00000000
5.00	7.83439230	0.61009019	0.00000000	0.00000000	0.00000000
7.50	7.82923260	0.91368333	0.00000000	0.00000000	0.00000000
10.00	7.82084090	1.21553720	0.00000000	0.00000000	0.00000000
12.50	7.81237820	1.51507730	0.00000000	0.00000000	0.00000000
15.00	7.80343510	1.81173330	0.00000000	0.00000000	0.00000000
17.50	7.78239040	2.10494060	0.00000000	0.00000000	0.00000000
20.00	7.74413660	2.39414100	0.00000000	0.00000000	0.00000000
22.50	7.67275790	2.67878400	0.00000000	0.00000000	0.00000000
25.00	7.46336310	2.95832780	0.00000000	-0.00000000	-0.00000000

AVERAGE = 0.00000001

RADIUS RP = 3.0000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)= .410(PINE), (SCAN-1)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.37462900	0.00000000	0.41000000	0.07795023	0.02283965
2.50	7.37400190	0.30533571	0.41000000	0.08012258	0.07022288
5.00	7.38398950	0.61009019	0.41000000	0.07943725	0.11544502
7.50	7.39079850	0.91368333	0.41000000	0.08112432	0.16380170
10.00	7.40549570	1.21553720	0.41000000	0.07813885	0.20078545
12.50	7.41997990	1.51507730	0.41000000	0.07995753	0.24793929
15.00	7.44718840	1.81173330	0.41000000	0.07809547	0.28175253
17.50	7.46737110	2.10494050	0.41000000	0.07883530	0.32225029
20.00	7.48549160	2.39414100	0.41000000	0.08025932	0.36408535
22.50	7.52617900	2.67878400	0.41000000	0.05493259	0.27219565
25.00	7.44424870	2.95832780	0.41000000	0.00703470	0.03747459

AVERAGE = 0.07497273

0.00000000000000000000

DISTANCE OF SOURCE D= 7.000000000

STARTING ANGLE TH= 0.000000000

STEP OF ANGLE DTH= 2.500000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION- $F(r) = .732$  (WALNUT), (SCAN-11)

TH	DATA	r	$F(r)$	$F(r)$	AREA PRODUCT
0.00	7.02642690	0.00000000	0.73200000	0.10599582	0.03104515
2.50	7.01031190	0.30533571	0.73200000	0.13385921	0.41731983
5.00	7.01481440	0.61009019	0.73200000	0.14407254	0.90938415
7.50	7.03174130	0.91368333	0.73200000	0.14556721	0.99392119
10.00	7.05703700	1.21553720	0.73200000	0.14687318	0.87740502
12.50	7.09589330	1.51507730	0.73200000	0.14815833	0.45939523
15.00	7.15617660	1.81173330	0.73200000	0.14429205	0.62057544
17.50	7.21890970	2.10494050	0.73200000	0.14091418	0.57500757
20.00	7.28482090	2.39414100	0.73200000	0.13957193	0.64315055
22.50	7.38956400	2.67878400	0.73200000	0.10935403	0.51141574
25.00	7.44132040	2.95832780	0.73200000	0.00923138	0.04017559

AVERAGE = 0.13674129

STEP 1: INPUT DATA 0.00000000

STEP 2: INPUT DATA 2.00000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)=1.000(WATER), ISCAN=1.0

TH	DATA	r	F(r)	FC(r)	AREA PRDCT
0.00	5.60665020	0.00000000	1.00000000	0.21591632	0.03353278
2.50	5.61338420	0.30533571	1.00000000	0.21504170	0.13947157
5.00	5.63987580	0.61009019	1.00000000	0.21164357	0.30758555
7.50	5.67203300	0.91368333	1.00000000	0.21035614	0.42473935
10.00	5.71659480	1.21553720	1.00000000	0.20862522	0.53608314
12.50	5.77650700	1.51507730	1.00000000	0.20789197	0.64455612
15.00	5.85751410	1.81173330	1.00000000	0.20554002	0.74151671
17.50	5.95081480	2.10494060	1.00000000	0.20163388	0.82420660
20.00	7.05012260	2.39414100	1.00000000	0.20451380	0.92775124
22.50	7.21450440	2.67878400	1.00000000	0.17483279	0.85552428
25.00	7.41637850	2.95832780	1.00000000	0.01881350	0.10022222

AVERAGE = 0.20091483

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)= .900(10%-VOID), (SCAN-1)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	5.79570580	0.00000000	0.00000000	0.16225525	0.04757593
2.50	5.79570580	0.30533571	0.00000000	0.16555010	0.44510384
5.00	5.80461450	0.61009019	0.00000000	0.17403460	0.05297215
7.50	5.82762930	0.91368333	0.00000000	0.17327777	0.84937281
10.00	5.85856500	1.21553720	0.00000000	0.17247345	0.44318745
12.50	5.89871450	1.51507730	0.00000000	0.17403907	0.63958565
15.00	5.94793710	1.81117330	0.00000000	0.18221703	0.65740208
17.50	7.02731450	2.10494060	0.00000000	0.17781935	0.72686129
20.00	7.10496540	2.39414100	0.00000000	0.18121389	0.02205105
22.50	7.22256600	2.67878400	0.00000000	0.16715309	0.02750524
25.00	7.39756160	2.95832780	0.00000000	0.02275501	0.42121861

AVERAGEI = 0.17502074



RADIUS R= 3.00000000  
 DISTANCE OF SOURCE D= 7.00000000  
 STARTING ANGLE TH= 0.00000000  
 STEP OF ANGLE DTH= 2.50000000  
 NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)= .800(20%-VOID), [SCAN-1]

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	5.90675480	0.00000000	0.80000000	0.15031400	0.04402552
2.50	5.91274290	0.30533571	0.80000000	0.14327039	0.12555862
5.00	5.92559520	0.61009019	0.80000000	0.14215797	0.20660151
7.50	5.93049480	0.91368333	0.80000000	0.15133259	0.30556323
10.00	5.95463890	1.21553720	0.80000000	0.15053619	0.33681747
12.50	5.98193470	1.51507730	0.80000000	0.15585451	0.43322520
15.00	7.02553830	1.81173330	0.80000000	0.16149930	0.53265571
17.50	7.08840880	2.10494050	0.80000000	0.15870599	0.61873289
20.00	7.13966040	2.39414100	0.80000000	0.16953455	0.75929967
22.50	7.23705910	2.67878400	0.80000000	0.16172950	0.80065560
25.00	7.39633530	2.95832780	0.80000000	0.02469450	0.03155050

AVERAGE = 0.15838280

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)= .700(30%-VOID), (SCAN-1)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.03526860	0.00000000	0.70000000	0.11943416	0.03499575
2.50	7.03790600	0.30533571	0.70000000	0.11495922	0.40075503
5.00	7.03966040	0.61009019	0.70000000	0.12471925	0.43125744
7.50	7.05444970	0.91368333	0.70000000	0.12103043	0.01447385
10.00	7.06561340	1.21553720	0.70000000	0.12147210	0.31213445
12.50	7.06987420	1.51507730	0.70000000	0.13459725	0.41731621
15.00	7.09754880	1.81173330	0.70000000	0.14390337	0.51917593
17.50	7.14598450	2.10494050	0.70000000	0.14363832	0.53714375
20.00	7.18083120	2.39414100	0.70000000	0.15835938	0.71842287
22.50	7.26542970	2.67878400	0.70000000	0.15251479	0.75503717
25.00	7.40610340	2.95832780	0.70000000	0.02191206	0.41668794

AVERAGE = 0.14078268

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)= .600(40%-VOID), (SCAV-1)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.20191630	0.00000000	0.60000000	0.09411783	0.02757499
2.50	7.20637730	0.30533571	0.60000000	0.08656477	0.07586900
5.00	7.21597500	0.61009019	0.60000000	0.08086662	0.11751082
7.50	7.20416930	0.91369333	0.60000000	0.09386559	0.13950783
10.00	7.21303160	1.21653720	0.60000000	0.08994829	0.23113094
12.50	7.20191630	1.51507730	0.60000000	0.10606403	0.32882005
15.00	7.21817680	1.81173330	0.60000000	0.11284404	0.43711833
17.50	7.23273310	2.10494060	0.60000000	0.12403809	0.53702300
20.00	7.25981960	2.39414100	0.60000000	0.13450400	0.61016077
22.50	7.31121840	2.67879400	0.60000000	0.13652157	0.67586191
25.00	7.41878090	2.95832780	0.60000000	0.01726653	0.09192810

AVERAGE = 0.11534759



RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)= .000(10%-VOID), LSCAV-2)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	5.81344460	0.00000000	0.00000000	0.17971734	0.05263748
2.50	5.82328610	0.30533571	0.00000000	0.16831558	0.44751893
5.00	5.83733280	0.61009019	0.00000000	0.17263458	0.05089432
7.50	5.86589110	0.91368333	0.00000000	0.16513039	0.83342197
10.00	5.88243750	1.21553720	0.00000000	0.17513931	0.43003762
12.50	5.92755790	1.51507730	0.00000000	0.17583024	0.54828927
15.00	5.99669150	1.81173330	0.00000000	0.17221157	0.62130429
17.50	7.06133440	2.10494050	0.00000000	0.17440815	0.71291763
20.00	7.13409380	2.39414100	0.00000000	0.18225200	0.82575336
22.50	7.26892010	2.67878400	0.00000000	0.16049315	0.79453470
25.00	7.45240250	2.95832780	0.00000000	0.00785883	0.04135489

AVERAGE = 0.17238833

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)= .800(20%-VOID), (SCAN-2)

TH	DATA	r	F(r)	F2(r)	AREA PRODUCT
0.00	5.94697600	0.00000000	0.80000000	0.14631443	0.04255409
2.50	5.95463890	0.30533571	0.80000000	0.13413016	0.41755720
5.00	5.95368420	0.61009019	0.80000000	0.15112427	0.01953244
7.50	5.98193470	0.91368333	0.80000000	0.14044777	0.03358429
10.00	5.99393300	1.21553720	0.80000000	0.14751720	0.07005901
12.50	7.02375890	1.51507730	0.80000000	0.15122101	0.05036008
15.00	7.06987420	1.81173330	0.80000000	0.15352777	0.55339451
17.50	7.11963560	2.10494060	0.80000000	0.15805932	0.64612713
20.00	7.17778240	2.39414100	0.80000000	0.16660007	0.75575034
22.50	7.28138570	2.67878400	0.80000000	0.15545470	0.77159154
25.00	7.45876270	2.95832780	0.80000000	0.00511035	0.02738324

AVERAGE = 0.15429374

RADIUS R= 7.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)= .700(30%-VOID), (SCAN-2)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.08002650	0.00000000	0.70000000	0.41675327	0.03419591
2.50	7.08422640	0.39533571	0.70000000	0.40950529	0.09597591
5.00	7.08673790	0.61009019	0.70000000	0.41685852	0.45984775
7.50	7.09257370	0.91368333	0.70000000	0.42175514	0.01584178
10.00	7.11069610	1.21553720	0.70000000	0.41950335	0.80707561
12.50	7.12769370	1.51507730	0.70000000	0.42324755	0.33212878
15.00	7.14834580	1.81173330	0.70000000	0.43364792	0.43217458
17.50	7.18690100	2.10494060	0.70000000	0.43545315	0.55781215
20.00	7.21450440	2.39414100	0.70000000	0.45399101	0.59856090
22.50	7.29709100	2.67878400	0.70000000	0.45004031	0.71278702
25.00	7.45991480	2.95832780	0.70000000	0.00366615	0.01953165
5.00	7.08673790	0.61009019	0.70000000	0.41685852	0.45984775
7.50	7.09257370	0.91368333	0.70000000	0.42175514	0.01584178
10.00	7.11069610	1.21553720	0.70000000	0.41950335	0.80707561
12.50	7.12769370	1.51507730	0.70000000	0.42324755	0.33212878
15.00	7.14834580	1.81173330	0.70000000	0.43364792	0.43217458
17.50	7.18690100	2.10494060	0.70000000	0.43545315	0.55781215
20.00	7.21450440	2.39414100	0.70000000	0.45399101	0.59856090
22.50	7.29709100	2.67878400	0.70000000	0.45004031	0.71278702

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)= .600(40%-VOID), [SCAN-2]

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.23489840	0.00000000	0.60000000	0.08345588	0.02444342
2.50	7.23849680	0.30533571	0.60000000	0.07448661	0.06528319
5.00	7.23633930	0.61009019	0.60000000	0.08080825	0.11744052
7.50	7.22911390	0.91368333	0.60000000	0.08963717	0.18099037
10.00	7.23417720	1.21553720	0.60000000	0.08873375	0.22801006
12.50	7.22983880	1.51507730	0.60000000	0.09875255	0.30618185
15.00	7.23489840	1.81173330	0.60000000	0.11065010	0.39923919
17.50	7.21636810	2.10494050	0.60000000	0.12320879	0.50363311
20.00	7.26892010	2.39414100	0.60000000	0.13779803	0.62510348
22.50	7.32974970	2.67878400	0.60000000	0.13857557	0.65602989
25.00	7.46679950	2.95832780	0.60000000	0.00200880	0.01070109

AVERAGE = 0.11407297

RADIUS R= 3.00000000  
 DISTANCE OF SOURCE D= 7.00000000  
 STARTING ANGLE TH= 0.00000000  
 STEP OF ANGLE DTH= 2.50000000  
 NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)= .000(10%-VOID), ISCAN-30

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	5.82654520	0.00000000	0.00000000	0.17461503	0.05114305
2.50	5.82871210	0.30533571	0.00000000	0.18001174	0.15776985
5.00	5.86171130	0.61009019	0.00000000	0.16392054	0.23822955
7.50	5.98038410	0.91368333	0.00000000	0.16385250	0.33084215
10.00	5.89779500	1.21553720	0.00000000	0.17375234	0.44549937
12.50	5.94985650	1.51507730	0.00000000	0.16811101	0.52122743
15.00	5.99942250	1.81173330	0.00000000	0.17414700	0.62828722
17.50	7.06475900	2.10494050	0.00000000	0.17835340	0.72904463
20.00	7.15383380	2.39414100	0.00000000	0.17774092	0.80629940
22.50	7.28207370	2.67878400	0.00000000	0.15825342	0.78344571
25.00	7.46336310	2.95832780	0.00000000	0.00635790	0.03392298

AVERAGE = 0.17359234



RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION  $F(r) = .800(20\% \text{-VOID})$ , [SCAN-3]

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	5.98193470	0.00000000	0.80000000	0.12526707	0.03668952
2.50	5.97821380	0.30533571	0.80000000	0.13255181	0.11617397
5.00	5.98379000	0.61009019	0.80000000	0.13952649	0.20277711
7.50	5.99209640	0.91368333	0.80000000	0.14795989	0.29875235
10.00	7.02820150	1.21553720	0.80000000	0.13712249	0.35234956
12.50	7.04925480	1.51507730	0.80000000	0.14253107	0.41207196
15.00	7.06002650	1.81173330	0.80000000	0.15209144	0.51871503
17.50	7.12367280	2.10494060	0.80000000	0.16137094	0.65952622
20.00	7.19293420	2.39414100	0.80000000	0.16515790	0.71926374
22.50	7.29979740	2.67878400	0.80000000	0.15270638	0.75598566
25.00	7.47250070	2.95832780	0.80000000	0.00284900	0.01517594

AVERAGE = 0.15139163

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION- $F(r) = .700(30\% \text{-VOID})$ , LSCAV-3J

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.13169850	0.00000000	0.70000000	0.10294429	0.03015139
2.50	7.13009850	0.30533571	0.70000000	0.10664910	0.09347175
5.00	7.13886700	0.61009019	0.70000000	0.10815594	0.15718553
7.50	7.15148550	0.91368333	0.70000000	0.10312184	0.20821785
10.00	7.15226890	1.21553720	0.70000000	0.10899714	0.25007879
12.50	7.15851400	1.51507730	0.70000000	0.11770725	0.35495079
15.00	7.17625460	1.81473330	0.70000000	0.12874340	0.45448004
17.50	7.21376830	2.10494060	0.70000000	0.13041104	0.53319588
20.00	7.23417720	2.39414100	0.70000000	0.15039425	0.63224468
22.50	7.31588350	2.67878400	0.70000000	0.14517138	0.71858300
25.00	7.46737110	2.95832780	0.70000000	0.00415240	0.02217360

AVERAGE = 0.12848704

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION  $F(r) = .600(40\% \text{ VOID})$ , (SCAN-3)

TH	DATA	r	F(r)	FC(r)	AREA PROJ
0.00	7.26262860	0.00000000	0.60000000	0.06533129	0.01913493
2.50	7.25205400	0.30533571	0.60000000	0.08245250	0.07225492
5.00	7.25122510	0.61009019	0.60000000	0.07834109	0.11385485
7.50	7.25911620	0.91368333	0.60000000	0.07977132	0.15105981
10.00	7.24422750	1.21553720	0.60000000	0.09413242	0.24201097
12.50	7.25205400	1.51507730	0.60000000	0.09880815	0.30535431
15.00	7.26682730	1.81173330	0.60000000	0.10452031	0.37708805
17.50	7.27378630	2.10494050	0.60000000	0.11814730	0.43294355
20.00	7.29437730	2.39414100	0.60000000	0.13178257	0.50781558
22.50	7.34601020	2.67878400	0.60000000	0.13561950	0.57139561
25.00	7.47703850	2.95832780	0.60000000	0.00240755	0.01232570

AVERAGE = 0.11071145



RADIUS R= 3.00000000  
 DISTANCE OF SOURCE D= 7.00000000  
 STARTING ANGLE TH= 0.00000000  
 STEP OF ANGLE DTH= 2.50000000  
 NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)= .000(10%-VOID), (SCAN-4)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
3.00	6.78219210	0.00000000	0.00000000	0.19180634	0.05617822
5.50	6.79682370	0.30533571	0.00000000	0.17224323	0.15095120
8.00	6.80903930	0.61009019	0.00000000	0.17840374	0.25927630
10.50	6.83195360	0.91368333	0.00000000	0.18000630	0.35345980
13.00	6.86901440	1.21553720	0.00000000	0.17517430	0.43783549
15.50	6.92165820	1.51507730	0.00000000	0.17345011	0.53778130
18.00	6.97541390	1.81173330	0.00000000	0.17849634	0.64398033
20.50	7.04838650	2.10494050	0.00000000	0.17795808	0.72745030
23.00	7.12769370	2.39414100	0.00000000	0.18281915	0.82933540
25.50	7.25911620	2.67878400	0.00000000	0.16385384	0.81122155
28.00	7.45356190	2.95832780	0.00000000	0.00565119	0.03010455

AVERAGE = 0.17594374

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)= .000(20%-VOID), (SCAT-4)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	5.94119010	0.00000000	0.80000000	0.14371951	0.04209109
2.50	5.94505110	0.30533571	0.80000000	0.13965455	0.42240797
5.00	5.95559260	0.61009019	0.80000000	0.14093498	0.20480577
7.50	5.95939850	0.91368333	0.80000000	0.15281085	0.30854715
10.00	5.99117690	1.21553720	0.80000000	0.14755632	0.37014303
12.50	7.02286810	1.51507730	0.80000000	0.14954500	0.45355375
15.00	7.05561340	1.81173330	0.80000000	0.15299260	0.55196599
17.50	7.11089610	2.10494050	0.80000000	0.16054643	0.65625592
20.00	7.17242460	2.39414100	0.80000000	0.16827392	0.75335354
22.50	7.28344820	2.67878400	0.80000000	0.15359721	0.75030579
25.00	7.45124170	2.95832780	0.80000000	0.00613330	0.03257282

AVERAGE = 0.15458759

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION- $F(r) = .700(30\% \text{-VOID})$ , ISCAJ-41

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.09837570	0.00000000	0.70000000	0.10523600	0.03082263
2.50	7.09872140	0.30533571	0.70000000	0.10874150	0.04530585
5.00	7.10742550	0.61009019	0.70000000	0.10447589	0.15163720
7.50	7.09754880	0.91368338	0.70000000	0.12154480	0.24541657
10.00	7.12205990	1.21553720	0.70000000	0.11285507	0.23997210
12.50	7.12929760	1.51507730	0.70000000	0.12061162	0.37395042
15.00	7.14203660	1.81173330	0.70000000	0.13503900	0.43757383
17.50	7.17548970	2.10494060	0.70000000	0.14353120	0.53670389
20.00	7.22402480	2.39414100	0.70000000	0.15095373	0.66482500
22.50	7.30384330	2.67878400	0.70000000	0.14684433	0.72695500
25.00	7.45876270	2.95832780	0.70000000	0.00341345	0.01834353

AVERAGE = 0.13369873

RADIUS = 0.70000000

DISTANCE OF SOURCE D= 7.00000000

SPREADING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION- $F(r) = .600(40\% \text{-VOID})$ , (SCAV-1)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.25981960	0.00000000	0.60000000	0.08490492	0.02486783
2.50	7.27100860	0.30533571	0.60000000	0.06025631	0.05282036
5.00	7.25417790	0.61009019	0.60000000	0.08288409	0.42045733
7.50	7.25981960	0.91368333	0.60000000	0.07965295	0.45083075
10.00	7.25347040	1.21553720	0.60000000	0.08645875	0.22218995
12.50	7.25700270	1.51507730	0.60000000	0.08937055	0.27709299
15.00	7.25134500	1.81173330	0.60000000	0.10493695	0.37859125
17.50	7.25559130	2.10494060	0.60000000	0.11994254	0.49028225
20.00	7.27655640	2.39414100	0.60000000	0.13345372	0.60544143
22.50	7.32909380	2.67878400	0.60000000	0.13633327	0.67492921
25.00	7.45472000	2.95832780	0.60000000	0.00362018	0.02993935

AVERAGE = 0.10938548

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION  $F(r) = .900(10\% \text{ VOID})$ , (SCAN-5)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	5.78671690	0.00000000	0.00000000	0.18537534	0.00129464
2.50	5.79570580	0.30533571	0.00000000	0.17601710	0.15425382
5.00	5.81124440	0.61009019	0.00000000	0.17951357	0.25089171
7.50	5.83518460	0.91368333	0.00000000	0.18132290	0.35511710
10.00	5.87935580	1.21553720	0.00000000	0.17281030	0.44051444
12.50	5.91869520	1.51507730	0.00000000	0.17725934	0.54952129
15.00	5.98471640	1.81173330	0.00000000	0.17425510	0.62871314
17.50	7.04403290	2.10494060	0.00000000	0.18210530	0.74433723
20.00	7.13727840	2.39414100	0.00000000	0.17953604	0.81455955
22.50	7.26052260	2.67873400	0.00000000	0.16123400	0.91330170
25.00	7.45529850	2.95832780	0.00000000	0.00563330	0.03550275

AVERAGE = 0.17559459



1.0113 RF= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)= .800(20%-VOID), (SCAN-5)

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	5.91373740	0.00000000	0.80000000	0.14220529	0.04165055
2.50	5.90675480	0.30533571	0.80000000	0.16052209	0.14058327
5.00	5.93439720	0.61009019	0.80000000	0.14672729	0.21324217
7.50	5.94215670	0.91368333	0.80000000	0.15553301	0.31114153
10.00	5.97073010	1.21553720	0.80000000	0.15434179	0.33659529
12.50	7.01031190	1.51507730	0.80000000	0.15245934	0.47257154
15.00	7.05531290	1.81173330	0.80000000	0.15497335	0.55911415
17.50	7.10002720	2.10494050	0.80000000	0.16314334	0.65687195
20.00	7.16006920	2.39414100	0.80000000	0.17359539	0.73749350
22.50	7.28069720	2.67878400	0.80000000	0.15607238	0.77251900
25.00	7.45529850	2.95832780	0.80000000	0.00617887	0.03291557

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)= .700(30%-VOID), ISCAN-50

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.04228620	0.00000000	0.70000000	0.12787930	0.03745453
2.50	7.05012260	0.30533571	0.70000000	0.11460420	0.10044393
5.00	7.05098940	0.61009019	0.70000000	0.12497200	0.43152561
7.50	7.06390400	0.91368333	0.70000000	0.12290800	0.971817007
10.00	7.06817200	1.21553720	0.70000000	0.13343530	0.31300355
12.50	7.10496540	1.51607730	0.70000000	0.12573670	0.39000121
15.00	7.11801620	1.81173330	0.70000000	0.14232990	0.51319753
17.50	7.18239750	2.10494050	0.70000000	0.14593950	0.57675302
20.00	7.20934030	2.39414100	0.70000000	0.15562490	0.90597293
22.50	7.29573510	2.67878400	0.70000000	0.14983910	0.71179257
25.00	7.45529850	2.95832780	0.70000000	0.00553550	0.02975570

AVERAGE = 0.14034507

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION-F(r)= .600(40%-VOID), ISCA 4-51

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.22037390	0.00000000	0.60000000	0.08853617	0.02594604
2.50	7.22183590	0.30533571	0.60000000	0.08562004	0.07504097
5.00	7.22911390	0.61009019	0.60000000	0.08343307	0.12125523
7.50	7.22402480	0.91368333	0.60000000	0.08992907	0.13157965
10.00	7.22256600	1.21553720	0.60000000	0.09493900	0.24395750
12.50	7.22183590	1.51507730	0.60000000	0.10537317	0.32983960
15.00	7.24565510	1.81173330	0.60000000	0.10814475	0.39016441
17.50	7.25205400	2.40494060	0.60000000	0.12417010	0.50756285
20.00	7.28619170	2.39414100	0.60000000	0.13115491	0.57495781
22.50	7.33367640	2.67878400	0.60000000	0.13541617	0.67533962
25.00	7.46106550	2.95832780	0.60000000	0.00523712	0.02771053

AVERAGE = 0.11441003



RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION- $F(r) = .000(N=N+SQRT(N))$

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.85448570	0.00000000	0.00000000	0.70452924	0.00534982
2.50	7.85252450	0.30533571	0.00000000	0.70569740	0.61850287
5.00	7.85409380	0.61009019	0.00000000	0.71937254	1.04548110
7.50	7.84898440	0.91368338	0.00000000	0.73453278	1.43322860
10.00	7.84067500	1.21558720	0.00000000	0.75549025	1.94130530
12.50	7.83229560	1.51507730	0.00000000	0.78523343	2.43735320
15.00	7.82344080	1.81173330	0.00000000	0.83112094	2.99851570
17.50	7.80260560	2.10494060	0.00000000	0.88910090	3.63432410
20.00	7.76473830	2.39414100	0.00000000	0.96557478	4.33474540
22.50	7.69410010	2.67878400	0.00000000	1.07023090	5.29825710
25.00	7.48703320	2.95832780	0.00000000	1.42105440	5.97204080

AVERAGEI = 0.37457583

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION- $F(r)=1.000(N=N+SQRT(N))$

TH	DATA	r	F(r)	FCH(r)	AREA PRODUCT
0.00	5.64275140	0.00000000	1.00000000	0.21381929	0.05262551
2.50	5.64936620	0.30533571	1.00000000	0.21195507	0.43577503
5.00	5.67539270	0.61009019	1.00000000	0.20863029	0.80327994
7.50	5.70699310	0.91368333	1.00000000	0.20744534	0.41885205
10.00	5.75079760	1.21553720	1.00000000	0.20577591	0.62875395
12.50	5.80971700	1.51507730	1.00000000	0.20405973	0.63268529
15.00	5.88942660	1.81173330	1.00000000	0.20287534	0.73193487
17.50	5.98129480	2.10494060	1.00000000	0.19910170	0.81385597
20.00	7.07914720	2.39414100	1.00000000	0.20200541	0.91637573
22.50	7.24126940	2.67878400	1.00000000	0.17275452	0.85523564
25.00	7.44060460	2.95832780	1.00000000	0.01853307	0.09899421

AVERAGE = 0.49320285

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION- $F(r) = .732(N+N\sqrt{N})$

TH	DATA	r	F(r)	F0(r)	AREA PRODUCT
0.00	7.05579240	0.00000000	0.73200000	0.40475408	0.03068145
2.50	7.03991160	0.30533571	0.73200000	0.43220209	0.41586741
5.00	7.04434840	0.61009019	0.73200000	0.44229338	0.20679902
7.50	7.05103000	0.91368333	0.73200000	0.44377745	0.29030742
10.00	7.08596300	1.21553720	0.73200000	0.44507490	0.37278419
12.50	7.12427040	1.51507730	0.73200000	0.44532345	0.45057472
15.00	7.18372280	1.81173330	0.73200000	0.44253595	0.51442120
17.50	7.24561650	2.10494050	0.73200000	0.43923007	0.56932672
20.00	7.31067300	2.39414100	0.73200000	0.43797133	0.62589193
22.50	7.41411300	2.67878400	0.73200000	0.40812791	0.53529524
25.00	7.46524980	2.95832780	0.73200000	0.00911480	0.04855562

AVERAGE = 0.43600305

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION- $F(r) = .410(N=N+\text{SQRT}(N))$

TH	DATA	r	F(r)	F(r)	AREA PRODUCT
0.00	7.39935980	0.00000000	0.41000000	0.07712071	0.02258791
2.50	7.39874040	0.30533571	0.41000000	0.07923079	0.05944119
5.00	7.40860620	0.61009019	0.41000000	0.07855502	0.11418033
7.50	7.41533260	0.91368333	0.41000000	0.08023825	0.15200251
10.00	7.42985230	1.21553720	0.41000000	0.07723340	0.19858725
12.50	7.44416290	1.51507730	0.41000000	0.07804353	0.24197361
15.00	7.47102870	1.81473330	0.41000000	0.07725238	0.27871092
17.50	7.49099440	2.10494050	0.41000000	0.07793655	0.31878094
20.00	7.50890430	2.39414100	0.41000000	0.07933930	0.35014210
22.50	7.54912560	2.67878400	0.41000000	0.05433314	0.25922813
25.00	7.46814350	2.95832730	0.41000000	0.00694129	0.03697705

AVERAGE = 0.07403847

RADIUS R= 30.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION- $F(r) = .000(N=N-SQRT(N))$

TH	DATA	r	F(r)	F2(r)	AREA PRODUCT
0.00	7.81469470	0.00000000	0.00000000	0.70127448	0.00539554
2.50	7.81269400	0.30533671	0.00000000	0.70235528	0.61558241
5.00	7.81429490	0.61009019	0.00000000	0.71507551	1.04058930
7.50	7.80908270	0.91363338	0.00000000	0.73120254	1.47540350
10.00	7.80060550	1.21553720	0.00000000	0.75195829	1.93225520
12.50	7.79205610	1.51507730	0.00000000	0.78260335	2.42545970
15.00	7.78302100	1.81173330	0.00000000	0.82723455	2.93457190
17.50	7.76175800	2.10494050	0.00000000	0.88497417	3.51745530
20.00	7.72310170	2.39414100	0.00000000	0.95194135	4.85372370
22.50	7.65095030	2.67878400	0.00000000	1.05453090	5.27029530
25.00	7.43911900	2.95832730	0.00000000	1.41093250	5.91805790

AVERAGE = 0.87035954



RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION  $F(r) = 1.000(N = N - \text{SORT}(N))$

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	5.56919680	0.00000000	1.00000000	0.22020832	0.05449590
2.50	5.57605910	0.30533571	1.00000000	0.21797042	0.49103842
5.00	5.50305100	0.61009019	1.00000000	0.21373905	0.81070464
7.50	5.63580620	0.91368333	1.00000000	0.21344320	0.43097271
10.00	5.58118060	1.21553720	1.00000000	0.21163853	0.64381311
12.50	5.74215610	1.51507730	1.00000000	0.21083654	0.65369464
15.00	5.82454940	1.81173330	1.00000000	0.20830335	0.75151525
17.50	5.91937650	2.10494060	1.00000000	0.20430245	0.83611475
20.00	7.02023020	2.39414100	1.00000000	0.20715158	0.93971717
22.50	7.18700340	2.67878400	1.00000000	0.17595425	0.87602680
25.00	7.39155100	2.95832780	1.00000000	0.01904435	0.40145420

AVERAGE = 0.20367303

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION- $F(r) = .732(N-N-SORT(N))$

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	5.99617270	0.00000000	0.73200000	0.40729834	0.03142580
2.50	5.97980930	0.30533571	0.73200000	0.43525252	0.41854983
5.00	5.93438140	0.61009019	0.73200000	0.44494555	0.021065278
7.50	7.00156870	0.91368333	0.73200000	0.44745103	0.029772489
10.00	7.02724930	1.21553720	0.73200000	0.44875555	0.03224197
12.50	7.06668720	1.51507730	0.73200000	0.45005542	0.45524514
15.00	7.12785020	1.81173830	0.73200000	0.4503833	0.52686044
17.50	7.19147010	2.10494050	0.73200000	0.44262908	0.53301625
20.00	7.25828270	2.39414100	0.73200000	0.44124878	0.64075741
22.50	7.36439700	2.67878400	0.73200000	0.41060392	0.64757365
25.00	7.41680440	2.95832780	0.73200000	0.00934298	0.04977115

AVERAGE = 0.43836802

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION- $F(r) = .410(N=N-SQRT(N))$

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.34927110	0.00000000	0.41000000	0.07887925	0.02310297
2.50	7.34863590	0.30533571	0.41000000	0.08071450	0.07074165
5.00	7.35875140	0.61009019	0.41000000	0.07934951	0.11532065
7.50	7.36564740	0.91363333	0.41000000	0.08205640	0.15568371
10.00	7.38053100	1.21553720	0.41000000	0.07902332	0.20305944
12.50	7.39519760	1.51507730	0.41000000	0.08083137	0.25077233
15.00	7.42272480	1.81173330	0.41000000	0.07892572	0.29475159
17.50	7.44317620	2.10494050	0.41000000	0.07972234	0.32587505
20.00	7.45151760	2.39414100	0.41000000	0.08115757	0.35820595
22.50	7.50269340	2.67873400	0.41000000	0.05555947	0.27505179
25.00	7.41976890	2.95832730	0.41000000	0.00712210	0.03794026

AVERAGE = 0.07374545



RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION- $F(r) = .000(N=N+3.0*SQRT(N))$

TH	DATA	r	F(r)	F(r)	AREA PRODUCT
0.00	7.39275380	0.00000000	0.00000000	0.70765935	0.00725963
2.50	7.39082900	0.30533571	0.00000000	0.70391000	0.62134353
5.00	7.39236920	0.61009019	0.00000000	0.72255492	1.05010500
7.50	7.38735490	0.91363333	0.00000000	0.73784432	1.43981510
10.00	7.37920060	1.21553720	0.00000000	0.75383779	1.95003660
12.50	7.37097830	1.51507730	0.00000000	0.78983034	2.44837320
15.00	7.36229010	1.81473330	0.00000000	0.83482375	3.01187180
17.50	7.34184990	2.10494050	0.00000000	0.89303414	3.65060590
20.00	7.30471030	2.39414100	0.00000000	0.97104301	4.40501490
22.50	7.73546480	2.67873400	0.00000000	1.07565430	5.32515330
25.00	7.53275630	2.95832730	0.00000000	1.43065390	5.02312580

AVERAGE = 0.87373099

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION- $F(r) = 1.000(N=N+(3.0*SQRT(N)))$

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	5.71126440	0.00000000	1.00000000	0.20814235	0.05095304
2.50	5.71766450	0.30533571	1.00000000	0.20532518	0.43083292
5.00	5.74285220	0.61009019	1.00000000	0.20323084	0.29537321
7.50	5.77344740	0.91368338	1.00000000	0.20209440	0.40805773
10.00	5.81588190	1.21553720	1.00000000	0.20048380	0.61515277
12.50	5.87300090	1.51507730	1.00000000	0.19997841	0.62003217
15.00	5.95035000	1.81173830	1.00000000	0.19794915	0.71415043
17.50	7.03960160	2.10494050	1.00000000	0.19435623	0.79445817
20.00	7.13478540	2.39414100	1.00000000	0.19734699	0.89523987
22.50	7.29274140	2.67878400	1.00000000	0.16889919	0.83614948
25.00	7.48736400	2.95832780	1.00000000	0.01813153	0.09685515

AVERAGE = 0.49361037

RADIUS R= 1.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION  $F(r) = .732(N=N+(3.0*SQRT(N)))$

TH	DATA	r	F(r)	FCI(r)	AREA PRODUCT
0.00	7.41205670	0.00000000	0.73200000	0.40243928	0.03000348
2.50	7.09660540	0.30533571	0.73200000	0.42912559	0.41317118
5.00	7.10092200	0.61009019	0.73200000	0.43899155	0.20199969
7.50	7.44715330	0.91358333	0.73200000	0.44045235	0.03359455
10.00	7.44141960	1.21553720	0.73200000	0.44163359	0.85405983
12.50	7.17871770	1.51507730	0.73200000	0.44303438	0.44347399
15.00	7.23663810	1.81173330	0.73200000	0.43940717	0.50295280
17.50	7.29698110	2.10494050	0.73200000	0.43513025	0.55665580
20.00	7.36045460	2.39414100	0.73200000	0.43493328	0.61233473
22.50	7.46147390	2.67878400	0.73200000	0.40580572	0.52880005
25.00	7.51145630	2.95832780	0.73200000	0.00892555	0.04754804

AVERAGE = 0.43210239

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION  $F(r) = .410(N=N+(3.0*SQRT(V)))$

TH	DATA	r	F(r)	F2(r)	AREA PRODUCT
0.00	7.44705900	0.00000000	0.41000000	0.07551400	0.02211544
2.50	7.44645370	0.30533571	0.41000000	0.07755154	0.05797319
5.00	7.45609310	0.61009019	0.41000000	0.07693479	0.41180577
7.50	7.46266570	0.91368333	0.41000000	0.07855378	0.45853160
10.00	7.47685500	1.21553720	0.41000000	0.07563051	0.49434000
12.50	7.49084220	1.51507730	0.41000000	0.07745504	0.04014899
15.00	7.51710650	1.81173830	0.41000000	0.07567122	0.02730064
17.50	7.53663000	2.10494050	0.41000000	0.07634447	0.01205855
20.00	7.55414690	2.39414100	0.41000000	0.07775950	0.05274521
22.50	7.59349710	2.67878400	0.41000000	0.05324873	0.025351229
25.00	7.51428540	2.95832780	0.41000000	0.00673554	0.03620119

AVERAGE = 0.07251347



RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION- $F(r) = .000(N=N-3.0*\text{SQRT}(N))$

TH	DATA	r	F(r)	F(r)	AREA PRODUCT
0.00	7.77325460	0.00000000	0.00000000	0.69789699	0.20440729
2.50	7.77121110	0.30533571	0.00000000	0.69890459	0.61254940
5.00	7.77284620	0.61009019	0.00000000	0.71265493	1.03671310
7.50	7.76752250	0.91368333	0.00000000	0.72769571	1.46932260
10.00	7.75886310	1.21553720	0.00000000	0.74331270	1.92285280
12.50	7.75012930	1.51507730	0.00000000	0.77373447	2.41461770
15.00	7.74089830	1.81473830	0.00000000	0.82330500	2.97031750
17.50	7.71917070	2.110494050	0.00000000	0.88069361	3.60995300
20.00	7.67965590	2.39414100	0.00000000	0.95713019	4.34191020
22.50	7.60585420	2.67873400	0.00000000	1.05869770	5.24117100
25.00	7.38879300	2.95832730	0.00000000	1.40019750	5.85038070

AVERAGE = 0.35510145

RADIUS RE 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION- $F(r)=1.000(N=N-(3.0*\text{SQRT}(r)))$

TH	DATA	r	F(r)	F(r)	AREA PROD.
0.00	5.48979930	0.00000000	1.00000000	0.22745853	0.05662041
2.50	5.49694990	0.33533571	1.00000000	0.22551530	0.49765107
5.00	5.52506500	0.61009019	1.00000000	0.22170158	0.82220409
7.50	5.55916070	0.91368338	1.00000000	0.22021828	0.44465245
10.00	5.50635200	1.21553720	1.00000000	0.21825594	0.65083003
12.50	5.66969790	1.51507730	1.00000000	0.21722109	0.67349285
15.00	5.75516940	1.81173330	1.00000000	0.21443192	0.77380725
17.50	5.85336990	2.10494060	1.00000000	0.21009645	0.85879853
20.00	5.95762340	2.39414100	1.00000000	0.21285523	0.95563542
22.50	7.12962290	2.67878400	1.00000000	0.18163951	0.89945903
25.00	7.30996640	2.95832780	1.00000000	0.01951337	0.40411249

AVERAGE = 0.29951307

RADIUS R= 3.00000000

DISTANCE OF SOURCE D= 7.00000000

STARTING ANGLE TH= 0.00000000

STEP OF ANGLE DTH= 2.50000000

NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION  $F(r) = .732(N-N-(3.0*\text{SQRT}(N)))$

TH	DATA	r	F(r)	F(r)	AREA PRODUCT
0.00	5.93277200	0.00000000	0.73200000	0.41011055	0.03225035
2.50	5.91586240	0.30533671	0.73200000	0.43933815	0.42215553
5.00	5.92058770	0.61009019	0.73200000	0.45000580	0.21800694
7.50	5.93834690	0.91368338	0.73200000	0.45153878	0.30596855
10.00	5.96487220	1.21553720	0.73200000	0.45285425	0.39279969
12.50	7.00558440	1.51507730	0.73200000	0.45403938	0.47775497
15.00	7.06867010	1.81173330	0.73200000	0.44995195	0.54099522
17.50	7.13422320	2.10494050	0.73200000	0.44532414	0.59812031
20.00	7.20299520	2.39414100	0.73200000	0.44485959	0.65713785
22.50	7.34207950	2.67878400	0.73200000	0.41344207	0.65150441
25.00	7.36589190	2.95832780	0.73200000	0.00953333	0.05105419

AVERAGE = 0.44209514

RADIUS R= 3.00000000  
 DISTANCE OF SOURCE D= 7.00000000  
 STARTING ANGLE TH= 0.00000000  
 STEP OF ANGLE DTH= 2.50000000  
 NUMBER OF ANNULUS RINGS N= 12

TABLE: DISTRIBUTION- $F(r) = .410(N=N-(3.0*\text{SQRT}(N)))$

TH	DATA	r	F(r)	FC(r)	AREA PRODUCT
0.00	7.29654050	0.00000000	0.41000000	0.08080912	0-02355321
2.50	7.29588790	0.30533571	0.41000000	0.08305154	0-07279862
5.00	7.30628010	0.61009019	0.41000000	0.08230539	0-01951853
7.50	7.31336380	0.91368333	0.41000000	0.08405655	0-05972229
10.00	7.32865020	1.21553720	0.41000000	0.08095318	0-02801713
12.50	7.34371070	1.51507730	0.41000000	0.08279255	0-05659793
15.00	7.37196860	1.81173330	0.41000000	0.08085470	0-02917432
17.50	7.39295600	2.10494050	0.41000000	0.08162218	0-03354205
20.00	7.41177310	2.39414100	0.41000000	0.08311550	0-07704303
22.50	7.45399980	2.67878400	0.41000000	0.05695304	0-03195073
25.00	7.36893480	2.95832780	0.41000000	0.00731417	0-03894743

AVERAGE = 0.07754893



THIS PROGRAM CALCULATES THE AVERAGE DISTRIBUTION  
 FOR SIMULATED DATA WITH RADIUS=1 AND THE DISTANCE  
 OF SOURCE FROM THE CENTRE=2.0. THIS PROGRAM FIRST  
 CALCULATES THE NUMERICAL INTEGRATION BY SIMPSON'S  
 1/3 RULE ALONG VARIOUS CHORDS SO WE HAVE THE DATA  
 VECTOR(I), THEN IT CALCULATES THE VARIABLES S(K,J)  
 FOR VARIOUS CHORDS AND THEN RECONSTRUCTS THE DISTRIBUTION  
 BY BACK SUBSTITUTION

```

INTEGER N
READ I0,I1,DAT(310),THH(310),S(310,310),F(310),FX(310)
READ I(310),AREA(310),ERR(310)
COMMON TH,D,E
READ(20,*)R,D,TH,DTH,S,I
THH(1)=0.0
THH(N+1)=(ASIN(R/D))*(180./3.141593)
TYPE *,THH(N+1)
DO 10 I=2,N
  THH(I)=THH(I-1)+DTH
  TYPE *,THH(I)
CONTINUE
10 T(1)=0.0
DO 20 I=1,N+1
  T(I)=(D*SIND(THH(I)))
CONTINUE
20 DO 25 I=1,N
  AREA(I)=3.141593*((T(I+1)**2)-(T(I)**2))
CONTINUE
25 SUMM1=0.0
DO 27 I=1,N
  SUMM1=SUMM1+AREA(I)
CONTINUE
27 DO 28 I=1,N
  R1=T(I)
  R1=R1+R1
  R2=T(I+1)
  R2=R2+R2
  RR=(R1+R2)/2.0
  RR=SQRT(RR)
  T(I)=RR
CONTINUE
28 DO 30 I=1,N
  F(I)=SQRT(1.0-(T(I)*T(I)))
  F(I)=1.0
CONTINUE
30 DO 40 I=1,N
  SINEB=(D*SIND(THH(I)))/R
  B=ASIN(SINEB)
  X1=-1.0*R*COS(B)
  X2=+1.0*R*COS(B)
  TH=THH(I)
  CALL INT(FF,X1,X2,I1)
  DAT(I)=I1
  TYPE *,I1
CONTINUE
40 L=1
DO 50 K=L,N
  DO 50 J=L,N
    S1=SIND(THH(J+1))
    S1=S1*S1
    S2=SIND(THH(I))

```

```

S2=S2+S2
S3=S1*(THH(1))
S3=S3+S3
Q1=S1-S3
Q2=S2-S3
Q1=SQRT(Q1)
Q2=SQRT(Q2)
Q=Q1-Q2
S(K,1)=2.0+Q*2
CONTINUE
L=L+1
50 CONTINUE
FX(N)=DAT(N)/S(N,N)
I=N-1
70 SUM=0.0
DO 30 J=I+1,N
SUM=SUM+(S(1,J)+FX(J))
80 CONTINUE
FX(I)=(DAT(I)-SUM)/S(1,I)
I=I-1
IF(I.LE.0)GO TO 70
DO 31 J=1,N+1
AREA(J)=AREA(J)+FX(J)
ERR(J)=F(J)-FX(J)
31 CONTINUE
SUMM2=0.0
DO 32 I=1,N
SUMM2=SUMM2+AREA(I)
32 CONTINUE
AVER=SUMM2/SUMM1
TYPE *,AVER
WRITE(21,100)R
100 FORMAT(1H, '//////////////////////////////////1X,3X,10HRADIUS R=,F12.8/)
WRITE(21,110)D
110 FORMAT(1X,3X,22HDISTANCE OF SOURCE D=,F12.8/)
WRITE(21,120)THH(1)
120 FORMAT(1X,3X,20HSTARTING. ANGLE TH=,F12.8/)
WRITE(21,130)DTH
130 FORMAT(1X,3X,19HSTEP OF ANGLE DTH=,F12.8/)
WRITE(21,140)E
140 FORMAT(1X,3X,24HERROR OF INTEGRATION E=,F12.8/)
WRITE(21,150)N
150 FORMAT(1X,3X,27HNUMBER OF ANNULUS RINGS N=,I6/)
WRITE(21,160)
160 FORMAT(1X,20X,'TABLE: DISTRIBUTION-F(r)= 1.000 '/')
WRITE(21,170)
170 FORMAT(1X, '-----')
1
WRITE(21,180)
180 FORMAT(1X,5X,'TH',9X,'DATA',13X,'r',11X,'F(r)',12X,'FCX(r)',8X,
180X,'ERRR',)
WRITE(21,190)
190 FORMAT(1X, '-----')
1
DO 200 I=1,N
WRITE(21,195)THH(I),DAT(I),T(I),F(I),FX(I),ERR(I)
195 FORMAT(1X,3X,F5.2,4X,F11.8,4X,F11.8,1X,F11.8,4X,F11.8,4X,F11.
200 CONTINUE
WRITE(21,210)AVER
210 FORMAT(1X, '-----')
210X,'AVERAGE DISTRIBUTION =',F10.8)
STOP

```

CH\*\*\*\*\*

```

SUBROUTINE INT(F,X1,Y2,I1)
  REAL I1,I2
  COMMON TH,D,E
  H=(X2-X1)/2.0
  I1=0.0
  X=X1
  CALL FUNC(F,X,TH,D)
  I2=0.0
  X=X2
  CALL FUNC(F,X,TH,D)
  S1=I1+I2
  S2=0.0
  X=X1+H
  CALL FUNC(F,X,TH,D)
  S4=0.0
  I1=(S1+4.0*S4)*(H/3.0)
  IF(I1.EQ.0.0)GO TO 5
  IF(ABS((I1-I2)/I1).LE.E)GO TO 6
  S2=S2+S4
  S4=0.0
  X=X1+(H/2.0)

```

```

DO 2 J=1,L
  CALL FUNC(F,X,TH,D)
  S4=S4+F
  X=X+H
2  CONTINUE
  H=H/2.0
  L=2*L
  I2=I1
  I1=(S1+2.0*S2+4.0*S4)*(H/3.0)
  GO TO 10
6  RETURN
END

```

CH\*\*\*\*\*

```

SUBROUTINE FUNC(F,X,TH,D)
  REAL F
  P1=0*SIND(TH)
  P2=P1*P1
  P3=X*X
  P4=P2+P3
  PA=SQRT(P4)
  F=1.0
  RETURN
END

```

20\*

THIS PROGRAM RECONSTRUCTS THE DISTRIBUTION

20\*

```

INTEGER N,CJDE,V1,V2,V3,1
READ 10,11,DA1(15),TH1(15),S(15,15),F(15),FX(15)
READ 10,11,DA(15),O3(15),ER(15)
READ 10,11,DA(15),O,TEMP,O1,O2,O3
COMMON TH,D,E,CJDE
READ(20,*)R,D,E,N,N1,N2,N3
F(1)=0.0
DO 1 I=2,N+1
  F(I)=F(I-1)+0.1
CONTINUE
THH(1)=0.0
DO 10 I=2,N+1
  SINTH=F(I)/D
  AN=ASIN(SINTH)
  THH(I)=(180.0*AN)/3.1415927
CONTINUE
R1=0.4
R2=0.8
R3=1.0
DO 11 I=1,N1
  F(I)=0.0
CONTINUE
DO 12 I=N1+1,N2+N1
  F(I)=1.0
CONTINUE
DO 13 I=N1+N2+1,N
  F(I)=2.0
CONTINUE
DO 14 I=1,N
  TH=THH(I)
  B=I
  SINB1=(D*SIND(THH(I)))/R1
  B1=ASIN(SINB1)
  SINB2=(D*SIND(THH(I)))/R2
  B2=ASIN(SINB2)
  SINB3=(D*SIND(THH(I)))/R3
  B3=ASIN(SINB3)
  IF(4.GT.N1)GO TO S1
  X1=0.0
  X2=1.0*R1*COS(B1)
  CODE=1
  CALL INT(FF,X1,X2,11)
  D1=2.0*I1
  CODE=2
  X1=1.0*R1*COS(B1)
  X2=1.0*R2*COS(B2)
  CALL INT(FF,X1,X2,11)
  D2=2.0*I1
  CODE=3
  X1=1.0*R2*COS(B2)
  X2=1.0*R3*COS(B3)
  CALL INT(FF,X1,X2,11)
  D3=2.0*I1
  GO TO 59
IF(4.GT.(N1+N2))GO TO S2
CJDE=2
X1=0.0
X2=1.0*R2*COS(B2)

```

```

CALLG INT(PF,X1,X2,I1)
D2=2.0*I1
CODE=3
X1=1.0*R2*COS(B2)
X2=1.0*R3*COS(B3)
CALLG INT(PF,X1,X2,I1)
D3=2.0*I1
GO TO 59
52 CODE=3
X1=0.0
X2=1.0*R3*COS(B3)
CALLG INT(PF,X1,X2,I1)
D3=2.0*I1
GO TO 59

54 IF(4.LE.N1)GO TO 35
IF(4.LE.(N1+N2))GO TO 36
DAT(M)=D3
GO TO 39
36 DAT(M)=D1+D2+D3
GO TO 39
36 DAT(M)=D2+D3
GO TO 39
39 M=I
40 CONTINUE

L=1
DO 50 K=L,N
    DO 50 J=L,N
        S1=SIND(THH(J+1))
        S1=S1*S1
        S2=SIND(THH(J))
        S2=S2*S2
        S3=SIND(THH(K))
        S3=S3*S3
        O1=S1-S3
        O2=S2-S3
        O1=SQRT(O1)
        O2=SQRT(O2)
        O=O1+O2
        S(K,J)=2.0*O*Q
    CONTINUE
    L=L+1
50 CONTINUE
60 FX(N)=DAT(N)/S(N,N)
I=N-1
SUM=0.0
70 DO 80 J=I+1,N
    SUM=SUM+(S(I,J)*FX(J))
80 CONTINUE
FX(I)=(DAT(I)-SUM)/S(I,I)
I=I-1
IF(1.LE.0)GO TO 70
DO 90 I=1,N
    ER(I)=F(I)-FX(I)
90 CONTINUE
WRITE(21,100)R
100 FORMAT(1X,3X,10HRADIUS R=,F12.6)
WRITE(21,110)D
110 FORMAT(1X,3X,22HDISTANCE OF SOURCE D=,F12.6)

```





```

C7D
*****
SUBROUTINE FUNCOP,X,11,0)
INTEGER CODE
READ,F
COMMON AA,BB,CC,CODE
10 GO TO (10,20,30),CODE
CONTINUE
F=0.0
20 GO TO 50
CONTINUE
F=1.0
30 GO TO 50
CONTINUE
F=2.0
50 RETURN
END

```

```

200 THIS PROGRAM CALCULATES THHS FOR ALL SCANS
210 I TO J, IT READS DATA FROM DATA FILE IS
220 FJRD.DAT AND THE OUTPUT FILE IS
230 FJRH.DAT

```

```

240
250 INTEGER J,CODE,N1,N2,N3,N
260 READ DAT(15),THH(15),S(15,15),F(15),FK(15)
270 READ JB(15),AREA(15),V,T(15)
280 COMMON TH,D,R,CODE
290 READ(2,*)R,D,TH,DTH,N,V
300 V=111
310 READ(2,*)(OR(I),I=1,N)
320 READ(2,*)(OAT(I),I=1,N)
330 DO 1 I=1,N
340 TYPE *,OB(I)
350 CONTINUE

```

```

360
370 DO 10 I=1,N
380 TEM=OAT(I)
390 DAT(I)=ALOG(TEMP)
400 TYPE *,DAT(I)
410 CONTINUE
420 THH(1)=0.0
430 THH(N+1)=ASIN(R/D)
440 THH(N+1)=(180.0*THH(N+1))/3.1415927
450 DO 20 I=2,N+1
460 THH(I)=THH(I-1)+DTH
470 CONTINUE
480 DO 30 I=1,N
490 F(I)=V
500 CONTINUE
510 DO 34 I=1,N
520 F(I)=(D*SIND(THH(I)))
530 CONTINUE
540 DO 39 I=1,N
550 AREA(I)=3.1415927*((T(I+1)**2)-(T(I)**2))
560 CONTINUE

```

```

570
580 L=1
590 DO 50 K=L,N
600 DO 50 J=L,N
610 S1=SIND(THH(J+1))
620 S1=S1*S1
630 S2=SIND(THH(J))
640 S2=S2*S2
650 S3=SIND(THH(K))
660 S3=S3*S3
670 O1=S1-S3
680 O2=S2-S3
690 O1=ABS(O1)
700 O2=ABS(O2)
710 Q=O1-O2
720 S(K,J)=2.0*D*Q
730 CONTINUE
740 L=L+1
750 CONTINUE
760 FK(N)=DAT(N)/S(N,N)
770 I=N-1

```



```

70 SUM=0.0
DO 80 J=1,N
SUM=SUM+(S(1,J)+FX(J))
80 CONTINUE
FX(1)=(DAT(1)-SUM)/S(1,1)
I=1-3
IF(LINE,3)GO TO 70
DO 91 K=1,N
TEMP=FX(K)-DB(K)
FX(K)=-TEMP
91 CONTINUE
DO 99 I=1,N
TEMP=FX(I)*AREA(I)
AREA(I)=TEMP
99 CONTINUE
SUMM1=0.0
DO 101 I=1,N-3
SUMM1=SUMM1+AREA(I)
101 CONTINUE
SUMM2=3.1415927*T(N-2)*I(N-2)
TYPE=*,SUMM2
AV=SUMM1/SUMM2
WRITE(21,100)R
100 FORMAT(1X,3X,10HRADIUS R=,F12.8/)
WRITE(21,110)D
110 FORMAT(1X,3X,22HDISTANCE OF SOURCE D=,F12.8/)
WRITE(21,120)THH(1)
120 FORMAT(1X,3X,20HSTARTING ANGLE TH=,F12.8/)
WRITE(21,130)DTH
130 FORMAT(1X,3X,19HSTEP OF ANGLE DTH=,F12.8/)
WRITE(21,140)E
140 FORMAT(1X,3X,24HERROR OF INTEGRATION E=,F12.8/)
N=N-1
WRITE(21,150)N
150 FORMAT(1X,3X,27HNUMBER OF ANNULUS RINGS N=,I4/)
N=N+1
WRITE(21,160)F(5)
160 FORMAT(1X,20X,'TABLE:DISTRIBUTION-F(r)=',F5.3/)
WRITE(21,170)
170 FORMAT(1X,
1-----
WRITE(21,180)
180 FORMAT(1X,5X,'TH',9X,'DATA',13X,'r',14X,'F(r)',12X,'FC(r)',8X
AREA PRODUCT',/)
WRITE(21,190)
190 FORMAT(1X,
1-----
DO 200 I=1,N-2
WRITE(21,195)THH(I),DAT(I),F(I),F(I),FX(I),AREA(I)
195 FORMAT(1X,3X,F5.2,4X,F11.8,4X,F11.8,4X,F11.8,4X,F11.8,4X,F11.
CONTINUE
WRITE(21,210)AV
210 FORMAT(1X,////////1X,25X,'AVERAGE =',F14.8)
STOP
END

```

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